Identifying the Primary Instructional Concepts in Mathematics: A Linguistic Approach

Robert J. Marzano

Requests for reprints should be sent to

Dr. Robert J. Marzano

7127 S Danube Court

Centennial, CO 80016

Abstract

Mathematics educators have been supplied with standards and benchmarks from multiple documents. However, these statements of expectations still require "unpacking" for teachers to ascertain the primary instructional concepts that might be the foci of specific lessons or units. The rationale for a linguistically-based process to identify these instructional concepts was presented along with the protocol for such an analysis. Using this protocol, 741 primary instructional concepts were identified from three sources of standards and benchmarks. These were organized into 52 categories. Finally, the 741 primary instructional concepts were rated by mathematics educators as to whether or not they are essential for all students to learn. Recommendations were made for utilization of this linguistic process for state-level standards documents.

One can make a strong case that mathematics educators have led the way in identifying what students should know and be able to do as a result of KB12 schooling. Indeed, the 1987 publication *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM) became the prototype document for standards documents in virtually every subject area (Marzano & Kendall, 1996). Since then, NCTM has updated the mathematics standards under the title of *Principles and Standards for School Mathematics* (NCTM, 2000). These documents are not the only ones articulating mathematics content. The National Research Council (NRC) has identified mathematics standards as part of a publication entitled *National Science Education Standards*. Additionally, 49 states have published mathematics standards documents. Finally, “composite” sets of standards B those incorporating standards from a variety of national and state documents – have been articulated by Mid-continent Research for Education and Learning (McREL) (Kendall & Marzano, 2000), and by the Council for Basic Education (CBE, 1998). In short, mathematics education stands at a unique point in time; one at which thousands of mathematics educators have spent a decade identifying and articulating the content of school mathematics.

However, even with all these resources, there is still a need for further clarification of the mathematics knowledge and skill articulated in the standards particularly from an instructional perspective. This is because classroom teachers must be fairly specific in the design of their lessons and units of instruction. However, the various standards documents cited above all articulate what students should know and be able to do in ways that require further “unpacking” if they are to be used as guidelines for classroom instruction. This need is evidenced by a simple examination of the benchmarks within various standards documents. Where a standard describes expectations about a general category of knowledge within a subject matter domain (i.e., mathematics or science), a benchmark more specifically describes the knowledge and skill students are expected to acquire at specific grade levels. To illustrate, below are five sample mathematics benchmarks for grade 5 (expectations for what students should know by the end of grade 5) – one from the CBE composite set of standards and benchmarks, two from the NCTM standards document, and two from the McREL composite set of standards and benchmarks:

CBE (p. 185)

*Use algorithms to solve addition, subtraction, multiplication, and division problems with whole numbers, fractions, and decimals.

NCTM (p. 392)

*Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

*Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to student's experience.

McREL (p. 51)

*Adds, subtracts, multiplies, and divides whole numbers and decimals.

*Adds and subtracts simple fractions.

There is certainly a great deal of overlap in these benchmarks, but the specifics of what the benchmarks have in common and what is unique to a given benchmark can be ascertained only after the benchmarks are decomposed. For example, the CBE benchmark includes the processes of addition, subtraction, multiplication, division, and the forms of whole numbers, fractions, and decimals in one statement. The CBE benchmark also identifies the use of algorithms as critical to these operations. The first benchmark from the NCTM document includes the processes of adding, subtracting, multiplying, and dividing in one statement along with whole numbers. It is the second benchmark that identifies fractions and decimals along with the processes of estimation and problem solving. Finally, the McREL document uses two statements to identify the processes of addition, subtraction, multiplication, division, and the formats of whole numbers, decimals, and fractions.

An important question not answered by the standards and benchmarks as articulated in the NCTM, CBE and McREL documents is what are the important conceptual units from an instructional perspective? Should teachers address the use of algorithms to solve addition, subtraction, multiplication, and division problems with whole numbers, fractions, and decimals as a single unit of instructional focus, as might be inferred if one were to adopt the CBE benchmark literally? Should teachers address strategies to estimate computations involving fractions and decimals in situations relevant to students' experience as a single unit of instructional focus, as might be inferred if one were to adopt the second NCTM benchmark literally, and so on. The most probable answer to these questions is no, although little guidance is provided in any of the documents. Perhaps, the discussion closest one comes to what might be interpreted as guidance relative to this issue is found in the NCTM (2000) document:

> In planning individual lessons, teachers should strive to organize the mathematics so that fundamental ideas form an integrated whole. Big ideas encountered in a variety of contexts should be established carefully, with important elements such as terminology, definitions, notation, concepts, and skills emerging in the process. (p. 15)

To implement this NCTM suggestion (or "curriculum principle" as described in the standards document) educators would necessarily have to unpack the benchmarks as articulated in the NCTM standards document (or the McREL or CBE documents). To this end, mathematics educators might be well served to have a simple listing of key instructional concepts that are addressed across the three documents such as:

addition of whole numbers
subtraction of whole numbers
multiplication of whole numbers
division of whole numbers
addition of decimals
subtraction of decimals
and so on.

Such a listing might provide an unambiguous and non-redundant accounting of the mathematics content around which teachers could plan instruction.

One purpose of the present study was to decompose the mathematics standards and benchmarks into basic units referred to here as “instructional concepts” that can be used by teachers to plan instruction.

This study also addressed the organization of these instructional concepts into meaningful instructional categories again in an effort to adhere to the NCTM curriculum principle of presenting mathematics concepts as an “integrated whole.” Although one might assume that the standards themselves represent these categories, this is not necessarily the case. In fact, the actual statements of standards in the various documents present mixed and somewhat confusing messages. To illustrate, the CBE document lists the benchmark discussed above within a standard entitled “Students will estimate and compute using mental math, estimation strategies, paper-and-pencil techniques, and technology-supported methods.” The NCTM document lists its benchmarks under a standard entitled “Numbers and Operations.” The McREL document lists the benchmarks under a standard entitled “Uses basic and advanced procedures while performing the processes of computation.”

Finally, this study sought to determine which of the instructional concepts (once identified) are considered truly essential for students to learn. The need for this endeavor becomes apparent when one examines the amount of content articulated in the standards documents. Specifically, the amount of content in the mathematics standards documents appears too great to be accommodated within the current context of K–12 education. For example, a study by Florian (1999) found that the amount of time estimated by teachers to adequately address the mathematics content articulated in the standards far exceeds the amount of time allocated for mathematics instruction. Similar findings have been reported by Marzano, Kendall, and Cicchinelli (1999).

One way to address the issue of too much content is to increase the amount of time available for instruction. While this is certainly a viable option, it is one that would require significant and costly changes in the structure of K–12 education, both of which render this option unlikely in the short run. Additionally, it must be noted that some research has indicated that simply increasing the amount of time available for instruction has little effect on student achievement (reported in Aronson, Zimmerman, & Carlos, 1999). Another way to address this issue is to make a distinction between those instructional concepts that are necessary for all students to learn as opposed to those concepts necessary for students seeking higher education. It is interesting to note that the content identified in the 2000 version of the NCTM standards is presented as necessary for all students. As noted in the document, the standards, "outline an ambitious and comprehensive set of curriculum standards for *all* [emphasis added] students" (p. 7). It is speculation only as to whether this represents a shift in policy from the 1989 NCTM standards document that made a distinction between the content critical for those students who intend to go on to college versus those who do not. For example, the 1989 document, under Standard 11 on probability, identified the following as content appropriate for students who intend to go to college: "apply the concept of random variable to generate and interpret probability distributions including binomial, uniform, normal, and chi square" (p. 171).

In summary, this study involved three basic research questions:

1. What are the primary instructional concepts articulated in the benchmarks of the NCTM, CBE, and McREL documents?
2. Into what categories can these instructional concepts be organized?
3. Which of the instructional concepts are considered essential for all students to learn?

In keeping with these research questions, the study involved three phases.

Phase One

The first phase of the study addressed the first research question. To answer this question, a linguistic analysis was performed on the benchmarks found in various documents articulating mathematics standards and benchmarks. The assumption underlying the use of this methodology was that the language used to express the content of the standards and benchmarks provides a window to the specific cognitive demands articulated in the standards and benchmarks. Stated differently, the validity of using a linguistic analysis to identify the specific instructional concepts explicit and implicit in the standards and benchmarks is grounded in the assumption that the manner in which language is used accurately portrays the intentions and thinking of the language user – in this case, those mathematics educators who constructed the standards and benchmarks. This assumption has a strong theoretical basis.

Arguably, the assertion that language mirrors human thought and intention became prominent in the late 1950s, when Noam Chomsky published *Syntactic Structures* (1957). There he posited the existence of an innate language mechanism. Chomsky expanded on and defended his theory in a number of later works (Chomsky, 1965, 1980, 1988). Relative to the present discussion, Chomsky's most powerful explanatory construct was his differentiation between two levels of linguistic structures – surface structures and deep structures. The surface structure of language deals with the actual use of language in oral or written form. The deep structure of language deals with the underlying semantic and syntactic structure of language. It is the deep structure of language that is used to form a representation of information in permanent memory (Kintsch, 1974; Kintsch & van Dijk, 1978). It is the deep structure of language, then, that provides a perspective on the cognitive demands of standards and benchmarks.

The actual format of deep structure representations of knowledge has been the subject of much discussion. The most popular model for describing the basic unit of thought within deep structure linguistic representations is the proposition. The construct of a proposition has a rich history in both psychology and linguistics (see Frederiksen, 1975; Kintsch, 1974; Norman and Rumelhart, 1975). In simple terms, "a proposition is the smallest unit of thought that can stand as a separate assertion, that is, the smallest unit about which it makes sense to make the judgment true or false" (Anderson, 1990, p. 123).

Case grammarians like Fillmore (1968) and Chafe (1970) and psychologists like Turner and Greene (1977) have described the various components of a proposition. To illustrate, consider the following proposition:

A circle is a type of figure

This proposition is stated in surface-level format (recalling Chomsky′s distinction between surface structures and deep structures). The deep structure version of this surface-level proposition might be represented as follows:

argument: *circle*

relation: *type of*

object: *figure*

As depicted above, propositions contain an argument, a relation, and an object (see Turner & Green, 1977). The argument is the concept about which information is provided. The object is the information that modifies or specifies the argument. The relation describes the type of relationship existing between the argument and the object. This implies that the argument within the deep structure format of a proposition is the "fulcrum" or "centerpiece" of a proposition to which all other elements of the proposition relate. One might say, then, that the concept of *circle* is the central concept in the proposition above.

Quite obviously, within propositional analysis, the order in which concepts are presented is critical. The assumption in this example is that the concept *circle* is mentioned first because it is the concept of primary emphasis in the proposition. Emphasis would have been quite different had the proposition been stated as: *One type of figure is a circle*. This quality of propositions is supported by at least two lines of evidence. First, a number of models of semantic memory assume that the argument in a proposition is the primary linkage between propositions (see Kintsch, 1974; Kintsch & van Dijk, 1978). Second, studies on information recall indicate that arguments within propositions are more readily retrieved than other elements (see Anderson & Bower, 1973; Bransford & Franks, 1971). In summary, relative to the task of identifying the instructional concepts as stated in the mathematics standards documents, the arguments within the deep structure propositions that underlie the benchmarks in those documents would provide a viable indication of their nature.

The basic purpose of this phase of the study, then, was to analyze the mathematics content as specified in selected standards documents from a linguistic perspective in an attempt to identify the primary instructional concepts. Wixson and Dutro (1999) have performed a linguistic analysis of selected standards documents in the language arts. However, this analysis was performed on surface-level language that was not parsed into a propositional format. To date, no linguistic analysis has been conducted on mathematics standards documents.

Method

Three mathematics standards documents were used to construct the deep structure propositional database from which primary instructional concepts were identified. They are: *Principles and Standards for School Mathematics* (NCTM, 2000), *Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education* (Kendall & Marzano, 2000), and *Standards for Excellence in Education* (CBE, 1998). *Principles and Standards for School Mathematics* is a revision of *Curriculum and Evaluation Standards for School Mathematics* published by NCTM in 1989. This earlier work contributed significantly to the national awareness of the benefits of identifying standards in content domains. *Content Knowledge* is a composite set of standards and benchmarks. The documents used to construct the mathematics standards and benchmarks within this source include: the NCTM (2000) document, *Benchmarks for Science Literacy* (Project 2061, 1993); *Mathematics Framework for the 1996 National Assessment of Educational Progress* (National Assessment of Educational Progress, n.d.); and from the New Standards Project: *Performance Standards: English, Language Arts, Mathematics, Science, Applied Learning* for the elementary, middle, and high school levels (New Standards, 1997a, 1997b, 1997c). *Standards for Excellence in Education* published by CBE is also a composite set of standards and benchmarks described as a "blend of standards from Delaware, New Jersey, and Pennsylvania" (p. 181) for the subject of mathematics.

Each benchmark statement in the three source documents was decomposed into surface structure propositions. Each surface structure proposition was then translated into its deep structure propositional format. To illustrate, consider the following surface structure proposition:

The area formula is l x w = a

As discussed previously, the deep structure format for this proposition would be:

argument:	*area formula*[1]
relation:	*is*
object:	*l x w = a*

The complete argument in the deep structure proposition was considered to be the primary instructional concept in the proposition. In summary, the protocol for identifying the primary instructional concepts as defined in this study (research question one) can be described as follows:

1. Decompose the benchmark statements in the source documents into surface structure propositions.
2. Merge the surface structure propositions from the source documents into a database of unique surface structure propositions.
3. Translate the surface structure propositions into their deep structure formats.
4. Consider the complete argument in the deep structure proposition to be the primary instructional concept for the proposition.

This protocol was executed by one researcher (the author). The reliability of steps one and three of the protocol were estimated. For step one of this protocol, two independent judges (the author being one) decomposed randomly selected benchmarks from one of the source documents. From this analysis, one rater produced 104 surface-level propositions, the other produced 101. An analysis of the content of these propositions indicated that, except for slight wording differences, 96 were identical. To estimate the reliability of step three of the protocol, two raters (the author being one) translated 100 of the surface strucutree propositions into their deep structure format. Except for minor wording differences, the raters agreed on 91 of 100 propositions.

Results

The first phase of this study addressed the identification of primary instructional concepts. In all, 741 unique, deep structure propositions were identified from the three sources of mathematics standards and benchmarks. From these propositions, 741 primary instructional concepts were extracted. These are reported in the second column of the appendix. The distribution of these concepts across four intervals of grade levels is presented in Table 1.

Table 1 Here

It is important to note that the primary instructional concepts identified using the linguistic analysis described above do not provide information as to the complexity of or density of knowledge implicit in a given concept. Consequently, the sheer number of primary instructional concepts at a given grade level interval is probably not a good indication of the conceptual load of the content at that grade level interval. In fact, the benchmarks at the lower grade levels tended to be far more explicit in their delineation of what students should know and be able to do. To illustrate, two of the instructional concepts identified at the K–2 grade level interval (Level 1) were *greater than* and *less than*. Contrast these with the following two instructional concepts identified at the 9–12 (Level 4) grade level interval: *polar coordinates* and *Cartesian coordinates*. Certainly a thorough coverage of the concept of *polar coordinates* or *Cartesian coordinates* would involve more information and higher levels of complexity than coverage of the concept *greater than* or *less than*. The tendency of the propositions at the lower grade levels to be more explicit than the propositions at higher grade levels most probably explains the fact that more instructional concepts were identified at the 3–5 grade level interval (216) than at the 6–8 grade level interval (205).

This fact notwithstanding, it is still interesting to note the sheer number of instructional concepts identified in this study – 741 in all. This implies that the conceptual load of mathematics represented in the standards documents is substantial. Stated differently, mastery of the content presented in the mathematics standards documents appears to be a daunting task for K–12 students. This inference is highly consistent with previous studies that have found the amount of time necessary to address the content articulated in mathematics standards documents far exceeds the amount of time available for mathematics instruction (Florian, 1999; Marzano et al, 1999).

Phase Two

The second phase of the study (research question two) dealt with the identification of instructional categories – groups of concepts that logically go together.

Method

This phase of the study was accomplished in two stages. In the first stage, a linguistic approach was again used to cluster the instructional concepts identified in phase one into meaningful categories. Specifically, the following protocol was used to provide an initial clustering of instructional concepts.

1. Terms or phrases indicating a type or category designation were deleted.

   e.g.: If *right angle* is the instructional concept, *right* is deleted and *angle* is considered the concept's category.

2. Terms or phrases indicating some aspect or feature of the instructional concept were deleted.

   e.g.: If *area formula* is the instructional concept, *formula* is deleted and *area* is considered the concept's category.

3. Instructional concepts that consistently appear in the same benchmarks were organized under the same category.

   e.g.: If the instructional concepts *length*, *width*, and *height* are always or consistently mentioned within the same benchmark, they are organized under the same category. In cases like this, the category was commonly assigned a compound label such as *length/width/height*.

The author performed this initial stage of categorization. As a result of this preliminary analysis, 50 categories were identified. In the second stage of this analysis, the 50 initial categories were reviewed independently by 10 mathematics educators. Their mean years of experience was 10.4 (SD = 2.6). Each educator was asked to: (1) re-categorize instructional concepts as they felt necessary, and (2) identify new categories or change or delete categories as necessary. If two or more of the mathematics educators re-categorized a concept into the same category, then the concept was re-categorized. If two or more mathematics educators identified a similar new category, that category was adopted. All contradictions in categorization by the mathematics educators were resolved by the author. In all, 47 concepts were re-categorized, two original category names were changed, two categories were deleted, and four categories were added for a final total of 52 categories.

Results

The final results of this analysis are reported in Table 2 and in column one of the appendix.

Table 2 Here

Table 2 depicts the distribution of the instructional categories across the four grade-level intervals. Although it is certainly true that the instructional concepts can be legitimately organized into categories different from those depicted in Table 2, the frequency distribution provides an interesting perspective on a possible scope and sequence of mathematics content. Specifically, taking Table 2 at face value, some conceptual categories should be addressed at specific levels only, whereas others span multiple grade-level intervals. For example, the category of *area* spans all grade level intervals as does the category of *number and number systems*. However, the category of *polynomials* is found at the 9–12 level only.

Phase Three

The final phase of this effort (research question three) addressed the perceived importance of the 741 instructional concepts. As described previously, the need for this delineation is justified by the sheer amount of content in the mathematics standards.

Method

Ten mathematics educators – the same 10 who were involved in phase two – were asked to identify those concepts that are "necessary for every person to understand to function well as literate adults regardless of whether they seek a college education." Specifically, for each instructional concept, each rater made a dichotomous determination as to whether or not it is necessary for *all* students to understand.

Results

The results of this phase of the study are reported in Table 3 and in column four of the appendix:

Table 3 Here

As depicted in Table 3, 299 instructional concepts were identified by all 10 mathematics educators as necessary for all students to know, 316 instructional concepts were identified by nine or more mathematics educators and so on. Of course, the criterion as to the percentage of mathematics educators who must identify an instructional concept as essential is arbitrary. However, if one accepts the intuitively appealing criterion of “a majority of mathematics educators” (i.e. six or more in the context of the current study), then 404 of the 741 instructional concepts identified in this study are necessary for all students to know prior to high school graduation. Whatever the appropriate criterion might be, these findings indicate that not all of the content in the mathematics standards is considered essential for students to learn. Indeed, 143 instructional concepts were not identified by any of the mathematics educators as essential. This finding seems to correspond to the recommendations from commentaries on the Third International Mathematics and Science Study (e.g., Stevenson & Stigler, 1992; Stigler & Hiebert, 1999) that the topics in U.S. mathematics curriculum should be reduced.

General Discussion

The limitations of this study are significant. Phase one employed a linguistic analysis of deep structure propositions to identify primary instructional concepts. The validity of such a process is dependent on the extent to which the language used to describe mathematics standards and benchmarks accurately portrays the comprehensive intent of those who designed the standards and benchmarks. Consequently, it is possible that some instructional concepts important to mathematics were not identified by this linguistic analysis because they were implicit in the language used to articulate standards and benchmarks. Phase two of this study sought to identify categories of instructional concepts that might provide guidance for curriculum sequencing. However, by its very nature, the process of categorizing is an indefinite one (see Smith & Medin, 1981). Consequently, it is reasonable to assume that the 741 instructional concepts identified in this study could be organized into a variety of equally valid categories. Phase three of the study sought to identify those instructional concepts that are considered essential by mathematics educators. However, the opinions of only 10 mathematics educators were used. This is certainly not a representative sample of mathematics educators across the country.

With these limitations noted, the findings of this study illuminate some significant issues hitherto not addressed in mathematics education. First, this study demonstrates that the standards and benchmarks articulated in standards documents can be linguistically unpacked into discernable instructional concepts that might be used by classroom teachers to plan lessons. Where this study utilized the NCTM (2000) standards and two sets of composite standards, individual states might wish to conduct linguistics analyses of their state documents in an effort to identify state-specific instructional concepts. Indeed, an effort like the one described here is probably most useful if conducted at the state level. This is most probably so because the standards movement in the United Stated is fundamentally a state-level effort. Indeed, state efforts to design standards were given an endorsement at the second education summit in Palisades, New York, in March 1996 when the state governors committed to designing standards and sharing conceptual and technical information regarding their efforts (National Governor's Association, 1996). These actions are consistent with the opinions of those educators who believe that it is at the state level that the standards movement will either succeed or fail. As education reporter Olson (1995) notes:

> The U.S. Constitution makes it clear: States bear the responsibility for educating their citizens. They decide how long students continue their education and how the schools are financed. They control what is taught, what is tested, which textbooks are used, and how teachers are trained.
>
> Thus, despite all the talk about national education standards, it is the 50 individual states that ultimately will determine what students should know and be able to do. (p.15)

Second, this study illustrates that instructional concepts can be organized into useful instructional categories that provide guidance in terms of the sequencing of instruction across grade level intervals. Again, if states were to identify state-specific instructional concepts, instructional categories might be identified initially by using the linguistically-based protocol described in phase two of this study, and then building on those results by using the judgments of mathematics educators.

Finally, and perhaps most importantly, this study demonstrates that there is probably not agreement that all the instructional concepts articulated in national-level standards documents and (perhaps) state-level documents are necessary for all students to master regardless of their post secondary educational aspirations. If states were to identify the instructional concepts explicit and implicit in their standards and benchmarks, they could undertake a study using a representative sample of mathematics educators (and/or other sectors of the population) to identify those concepts considered essential for all students versus those considered necessary only for those seeking post secondary educations. This might go a long way in reducing the number of instructional concepts to a level that can be adequately addressed in the instructional time available to mathematics educators.

References

Anderson, J. R. (1990). *Cognitive psychology and its implications* (3rd ed.). New York: W. H. Freeman.

Anderson, J. R., & Bower, G. H. (1973). *Human associative memory*. Washington, DC: Winston.

Aronson, J. Zimmerman, J. & Carlos, L. (1999). *Improving student achievement by extending school: Is it just a matter of time?* San Francisco, CA: WestEd.

Bransford, J. D., & Franks, J. J. (1971). The abstraction of linguistic ideas. *Cognitive Psychology, 2,* 331-380.

Chafe, W. L. (1970). *Meaning and structure of language.* Chicago: University of Chicago Press.

Chomsky, N. (1957). *Syntactic structures.* The Hague: Moutan.

Chomsky, N. (1965). *Aspects of a theory of syntax.* Cambridge, MA: MIT Press.

Chomsky, N. (1980). Rules and representations. *Behavioral and Brain Science, 3,* 1-61.

Chomsky, N. (1988). *Language and the problems of knowledge: The Managua lectures.* Cambridge, MA: MIT Press.

Council for Basic Education. (1998). *Standards for excellence in education.* Washington, DC: Author.

Fillmore, C. J. (1968). The case for case. In E. Beck & R. T. Harms (Eds.), *Universals in linguistic theory* (pp. 1-210). New York: Holt, Rinehart & Winston.

Florian, J. (1999). *Teacher survey of standards-based instruction: Addressing time.* Aurora, CO: Mid-continent Research for Education and Learning.

Frederiksen, C. H. (1975). Representing logical and semantic structure of knowledge acquired from discourse. *Cognitive Psychology*, *7,* 371-458.

Kendall, J. S., & Marzano, R. J. (2000). *Content knowledge: A compendium of standards for KB12 education* (3rd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

Kintsch, W. (1974). *The representation of meaning in memory*. Hillsdale, NJ: Erlbaum.

Kintsch, W., & van Dijk, T. A. (1978). Towards a model of text comprehension and production. *Psychological Review*, *85,* 363-394.

Marzano, R. J., & Kendall, J. S. (1996). *A comprehensive guide to designing standards- based* *districts, schools, and classrooms.* Alexandria, VA: Association for Supervision and Curriculum Development.

Marzano, R. J., Kendall, J. S., & Cicchinelli, L. (1999). *What Americans believe students should know: A survey of U.S. adults.* Aurora, CO: Mid-continent Research for Education and Learning.

National Assessment of Educational Progress. (n.d.). *Mathematics framework for the 1996 National Assessment of Educational Progress.* Washington, DC: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Governor's Association (1996, March). *1996 national education summit policy statement*. Washington, DC: Author.

National Research Council. (1996). *National science education standards.* Washington, DC: National Academy Press.

New Standards. (1997a). *Performance standards: English language arts, mathematics, science, applied learning, volume 1, elementary school.* Washington, DC: National Center on Education and the Economy.

New Standards. (1997b). *Performance standards: English language arts, mathematics, science, applied learning, volume 2, middle school.* Washington, DC: National Center on Education and the Economy.

New Standards. (1997c). *Performance standards: English language arts, mathematics, science, applied learning, volume 3, high school.* Washington, DC: National Center on Education and the Economy.

Norman, D. A., & Rumelhart, D. E. (1975). *Explanations in cognition.* New York: W. H. Freeman and Company.

Olson, L. (1995, April 12). Standards times 50. *Struggling for standards: An Education Week Special Report*, 14-22.

Project 2061, American Association for the Advancement of Science. (1993). *Benchmarks for science literacy*. New York: Oxford University Press.

Smith, E.E., & Medin, D.L. (1981). *Categories and concepts.* Cambridge, MA: Harvard

University Press.

Stevenson, H.W., & Stigler, J.W. (1992). *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education*. New York: Simon and Schuster.

Stigler, J.W. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Turner, A., & Greene, E. (1977). *The construction of a propositional text base*. Boulder, CO: Institute for the Study of Intellectual Behavior, The University of Colorado at Boulder.

Wixson, K. K., & Dutro, E. (1999). Standards for primary-grade reading: An analysis of state frameworks. *The Elementary School Journal*, *100*(2), 89-110.

Appendix

Table A1

CATEGORY	CONCEPT	LEVEL	MASTER
addition	addition (gen characteristics)	1	10
	addition facts	1	10
	addition whole numbers	1	10
	sum	1	10
	addend	2	5
	associative property addition	2	5
	commutative property addition	2	5
	addition algorithms	2	10
area	area (gen chars)	1	10
	parallelogram formula	2	5
	rectangle formula	2	5
	triangle formula	2	5
	area measures (gen chars)	2	10
	conservation of area	2	10
	surface area (concept of)	2	10
	trapezoid formula	3	0
	circle formula	3	8

CATEGORY	CONCEPT	LEVEL	MASTER
	area of irregular shapes	3	10
	surface area cone	4	0
	surface area cylinder	4	0
	surface area sphere	4	0
central tendency & variability	median	2	6
	center measures (gen chars)	2	10
	cluster	2	10
	extreme	2	10
	dispersion (gen chars)	3	8
	mean	3	10
	midpoint	3	10
	mode	3	10
	range	3	10
	central limit theorem	4	0
	quartile deviation	4	0
	sigma notation	4	0
	variance	4	2
	standard deviation	4	4
charts & graphs	graphs (gen chars)	1	10
	bar graphs	2	10

CATEGORY	CONCEPT	LEVEL	MASTER
	histograms	2	10
	line graphs	2	10
	pie charts	2	10
	box & whisker plots	3	8
	stem & leaf plots	3	8
	grids	3	10
	ordered pairs	3	10
	scatter plots	3	10
	parallel box plots	4	0
	Cartesian coordinates	4	5
	finite graphs	4	5
	two way tables	4	5
computation (general)	greater than	1	10
	less than	1	10
	associative property addition	2	5
	associative property subtraction	2	5
	commutative property addition	2	5
	commutative property subtraction	2	5
	identity (gen chars)	2	5
	reversing order of operations	2	9

CATEGORY	CONCEPT	LEVEL	MASTER
	order operations (gen chars)	2	10
	relative magnitude	2	10
	associative prop division	3	5
	associative prop multiplication	3	5
	commutative prop division	3	5
	commutative prop multiplication	3	5
	identity prop addition	3	5
	identity prop multiplication	3	5
	identity property (gen chars)	3	5
	absolute value	4	8
coordinate systems	2dim vs. 3dim systems	2	10
	coordinate systems (gen chars)	2	10
	horizontal axis	2	10
	number pairs	2	10
	number triplets	2	10
	vertical axis	2	10
	rectangular coordinates	3	0
	coordinate planes	3	4
	coordinate geometry	3	10
	polar coordinates	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	Cartesian coordinates	4	7
data collect & samples	data (gen chars)	1	10
	data sets	1	10
	data collect methods	2	10
	large samples	2	10
	sample (gen chars)	2	10
	surveys	2	10
	tallies	2	10
	sample space	3	1
	limited sample	3	5
	sample selection techniques	3	6
	biased sample	3	10
	data display errors	3	10
	random sample	3	10
	sampling error	3	10
	spreadsheets	3	10
	sampling distribution	4	2
	random sampling techniques	4	3
	population	4	4
	law of large numbers	4	5

CATEGORY	CONCEPT	LEVEL	MASTER
	representativeness of sample	4	5
data distributions	nominal data	2	6
	data gaps	2	9
	data clusters	2	10
	data extremes	2	10
	outliers	3	8
	frequency	3	10
	frequency distribution	3	10
	relative frequency	3	10
	bivariate data transformations	4	0
	bivariate distribution	4	0
	univariate distribution	4	0
	bivariate data	4	1
	categorical data	4	1
	probability distribution	4	1
	sampling distribution	4	1
	univariate data	4	1
	frequency distribution	4	3
	normal curve	4	7
decimals	decimal addition	2	10

CATEGORY	CONCEPT	LEVEL	MASTER
	decimal division	2	10
	decimal multiplication	2	10
	decimal subtraction	2	10
	decimals (gen chars)	2	10
	equivalent forms	2	10
	calculator use decimals	3	10
	decimal metal addition	3	10
	decimal estimation	3	10
direction position	above	1	10
location	behind	1	10
	below	1	10
	between	1	10
	direction (gen chars)	1	10
	in front	1	10
	inside	1	10
	left	1	10
	location (gen chars)	1	10
	orientation	1	10
	outside	1	10
	right	1	10

CATEGORY	CONCEPT	LEVEL	MASTER
	under	1	10
division	quotient	2	7
	fraction division	2	8
	decimal division	2	10
	division (gen chars)	2	10
	remainder	2	10
	whole number division	2	10
	associative prop division	3	5
	commutative prop divide	3	5
equations & inequalities	inequality solutions	2	4
	open sentence	2	6
	number sentence	2	7
	inequality (gen chars)	2	9
	equations	2	10
	quadratic equation	3	2
	slope intercept form	3	2
	formula missing values	3	4
	graphic solutions	3	4
	nonlinear equations	3	4
	equation systems	3	5

CATEGORY	CONCEPT	LEVEL	MASTER
	linear equations	3	5
	variable change	3	5
	substitution for unknowns	3	7
	place holder	3	8
	slope	3	10
	substitution (gen chars)	3	10
	unknown	3	10
	variables (gen chars)	3	10
	equivalent forms inequalities	4	0
	parametric equations	4	0
	recursive equations	4	0
	systems of inequalities	4	0
	equivalent forms of equations	4	2
estimation	estimation (concept of)	1	5
	concept of near	1	10
	front end digits	1	10
	part to whole	2	5
	front end estimation	2	7
	truncation	2	8
	estimation use (gen chars)	2	10

CATEGORY	CONCEPT	LEVEL	MASTER
	rounding	2	10
	whole number comp estimation	2	10
	range of estimations	3	5
	overestimation	3	8
	underestimation	3	8
	precision of estimation	4	6
experiments	investigations	2	10
	studies	2	10
	experiments	3	10
	control group	4	4
	experimental design	4	4
	treatment group	4	4
	statistical experiment	4	5
exponents logs roots	cube numbers	3	3
	scientific notation	3	4
	exponential notation	3	5
	exponents (gen chars)	3	8
	roots	3	8
	cube roots	3	9
	base	3	10

CATEGORY	CONCEPT	LEVEL	MASTER
	square roots	3	10
	base e	4	0
	exponentiation	4	0
	exponents & real numbers	4	0
	log functions	4	0
	natural logs	4	0
	roots & real numbers	4	0
	roots to determine profits	4	0
	roots to determine cost	4	0
	roots to determine revenue	4	0
	exponential functions	4	1
	negative exponents	4	1
	binary system	4	2
	logarithms (gen chars)	4	3
	powers	4	6
expressions	algebraic expression expansion	3	2
	algebraic expressions	3	4
	combining like terms	3	4
	math expressions (gen chars)	3	8
	addition radical expressions	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	division radical expressions	4	0
	multiplication radical expressions	4	0
	radical expressions	4	0
	subtraction radical expressions	4	0
	simplification	4	1
	term	4	2
factors multiples & primes	prime factorization	2	2
	greatest common factor	2	5
	least common multiple	2	5
	dividend	2	7
	divisibility	2	10
figures & shapes	2 dim shape combination	1	8
	2 dim shape decomposition	1	8
	3 dim shape combination	1	8
	2 dim shape	1	1
	3 dim shape	1	10
	circle	1	10
	corner	1	10
	object	1	10
	rectangle	1	10

CATEGORY	CONCEPT	LEVEL	MASTER
	square	1	10
	triangle	1	10
	rectangular prism	2	0
	rhombus	2	0
	isosceles triangle	2	2
	geometric properties	2	4
	intersection of shapes	2	4
	number of faces	2	4
	classes of triangles	2	5
	parallelogram	2	5
	shape division	2	5
	corresponding sides	2	6
	shape transformation	2	6
	equilateral triangle	2	7
	prism	2	8
	plane	2	9
	2 dim space	2	10
	3 dim space	2	10
	cubes	2	10
	cylinder	2	10

CATEGORY	CONCEPT	LEVEL	MASTER
	face	2	10
	perspective	2	10
	pyramid	2	10
	sphere	2	10
	tetrahedron	3	0
	plane figures	3	3
	polygon	3	3
	irregular polygon	3	4
	planar cross section	3	4
	quadrilateral	3	4
	solid figures	3	7
	triangle sides	3	7
	defining properties shapes/figures	3	8
	parallel figures	3	8
	arc	4	0
	chord	4	0
	circle without center	4	0
	isometry	4	0
	synthetic geometry	4	0
	transversal	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	3 dim cross section	4	1
fractions	subdivision	1	4
	unlike denominators	2	8
	improper fraction	2	9
	reduced form	2	9
	common denominator	2	10
	common fractions	2	10
	diff size fractions	2	10
	equivalent fractions	2	10
	estimation of fractions	2	10
	fraction addition	2	10
	fraction division	2	10
	fraction multiplication	2	10
	fraction subtraction	2	10
	fractions (gen chars)	2	10
	relative magnitude of fractions	2	10
	add fractions mental	3	8
	divide fractions mental	3	8
	multiply fractions mental	3	8
	subtract fractions mental	3	8

CATEGORY	CONCEPT	LEVEL	MASTER
	fraction inversion	4	3
functions	function (gen chars)	2	10
	algebraic step function	3	0
	approximate lines rep	3	0
	constant difference	3	0
	constant ratio	3	0
	vertex	3	1
	input/output table representation	3	2
	intercept	3	2
	graphic rep of functions	3	4
	nonlinear functions	3	4
	table rep of functions	3	4
	maximum	3	10
	minimum	3	10
	Venn diagram rep	3	10
	absolute function	4	0
	asymptote of function	4	0
	circular functions	4	0
	classes of functions	4	0
	curve fitting median method	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	direct functions	4	0
	domain of function	4	0
	exponential functions	4	0
	function composition	4	0
	function notation	4	0
	geometric functions	4	0
	global/local behavior	4	0
	inflection	4	0
	inverse functions	4	0
	linear translations	4	0
	logarithmic functions	4	0
	periodic functions	4	0
	phase shift	4	0
	polynomial functions	4	0
	radical functions	4	0
	range of functions	4	0
	rational functions	4	0
	sinusoidal functions	4	0
	step functions	4	0
	vertex edge graph	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	algebraic functions	4	1
	continuity	4	1
	curve fitting (gen chars)	4	1
	minimum/maximum of functions	4	1
	area under curve	4	2
	real world functions	4	3
length width height	distance (gen chars)	1	10
	size	1	10
	estimation of height	2	10
	estimation of length	2	10
	estimation of width	2	10
	height	2	10
	length	2	10
	width	2	10
	measure height	3	10
	measure length	3	10
	measure width	3	10
lines & angles	alternate interior angles	2	2
	angle measure tools	2	4
	complementary angles	2	4

CATEGORY	CONCEPT	LEVEL	MASTER
	corresponding angles	2	4
	supplementary angle	2	5
	angle units	2	6
	acute angles	2	7
	obtuse angles	2	7
	angle (gen chars)	2	10
	parallel lines	2	10
	perpendicular lines	2	10
	right angles	2	10
	angle bisectors	3	4
	perpendicular bisector	3	4
	line symmetry	3	5
	intersecting lines	3	10
	angle of depression	4	0
	central angle	4	0
	line segment congruence	4	0
	line segment similarity	4	0
	line segments	4	0
	line through point not on line	4	0
	line equations	4	4

CATEGORY	CONCEPT	LEVEL	MASTER
math reasoning	asking questions	1	10
	making conjectures	1	10
	logic ALL	2	6
	logic AND	2	6
	logic NONE	2	6
	logic NOT	2	6
	logic OR	2	6
	logic SOME	2	6
	verification	2	9
	counter examples	2	10
	deductive predictions	2	10
	invalid argument	2	10
	valid argument	2	10
	conjecture	3	7
	deductive arguments	3	10
	inductive arguments	3	10
	inductive reasoning	3	10
	formal mathematical induction	4	0
	incidence	4	0
	empirical verification	4	1

CATEGORY	CONCEPT	LEVEL	MASTER
	derivation	4	5
	nature of deduction	4	5
	postulate	4	5
	logic IF/THEN	4	6
	theories	4	7
matrices & vectors	arrays	3	7
	matrix addition	4	0
	matrix division	4	0
	matrix equations	4	0
	matrix inversion	4	0
	matrix multiplication	4	0
	matrix subtraction	4	0
	scalar	4	0
	vector addition	4	0
	vector division	4	0
	vector multiplication	4	0
	vector subtraction	4	0
	vector (gen chars)	4	1
	matrices (gen chars)	4	5
measurement	measuring cup	1	10

CATEGORY	CONCEPT	LEVEL	MASTER
	benchmarking	2	5
	reliability	2	8
	foot	2	10
	inch	2	10
	measurement (gen chars)	2	10
	ruler	2	10
	straight edge & compass	3	2
	convert large to small	3	10
	convert small to large	3	10
	precision of	3	10
	significant digits	3	10
	thermometer	3	10
	successive approximations	4	1
	unit analysis	4	4
	limits	4	5
	unit conversion	4	5
	upper/lower bounds	4	5
	protractor	4	7
	direct measure	4	8
	indirect measure	4	8

CATEGORY	CONCEPT	LEVEL	MASTER
	decibels	4	10
	Richter scale	4	10
	U.S. customary system	4	10
metric system	centimeters	2	10
	English system	2	10
	meters	2	10
	metric system (gen chars)	2	10
	convert large to small	3	10
	convert small to large	3	10
money	coins	1	10
	money (gen chars)	1	10
motion geometry	flip transformations	1	7
	shape division	1	7
	slide transformations	1	7
	shape combination	1	8
	2 dim slide	2	4
	2 dim turn	2	4
	movement (gen chars)	2	10
	rotation	2	10
	dilation	3	0

CATEGORY	CONCEPT	LEVEL	MASTER
	enlarging transformations	3	0
	reflection transformations	3	0
	rotation symmetry	3	0
	tessellation	3	0
	projection	3	1
	shrinking transformations	3	1
	dilation of object in plane	4	0
	reflection in plane	4	0
	reflection in space	4	0
	rotation in plane	4	0
multiplication	distributive property	2	5
	basic number combinations	2	10
	factors	2	10
	multiples	2	10
	multiplication (gen chars)	2	10
	product	2	10
	square numbers	3	2
	associative prop multiplication	3	5
	commutative prop multiplication	3	5
	multiplication algorithms	3	5

CATEGORY	CONCEPT	LEVEL	MASTER
	prime factors	3	5
numbers & number systems	cardinal numbers	1	10
	grouping	1	10
	number line	1	10
	numbers (gen chars)	1	10
	numerals (gen chars)	1	10
	ordinal numbers	1	10
	whole numbers	1	10
	expanded notation	2	1
	mixed numbers	2	7
	even numbers	2	10
	negative numbers	2	10
	odd numbers	2	10
	positive numbers	2	10
	prime numbers	2	10
	zero	2	10
	relatively prime	3	0
	composite numbers	3	4
	non-decimal numeration system	3	4
	number property	3	4

CATEGORY	CONCEPT	LEVEL	MASTER
	number systems	3	4
	number theory	3	4
	rational numbers	3	4
	subsets	3	9
	base 60	3	10
	integers	3	10
	large numbers	3	10
	Roman numerals	3	10
	conjugate numbers	4	0
	natural numbers	4	0
	number subsystems	4	0
	irrational numbers	4	2
	real numbers	4	2
	imaginary number	4	4
	complex numbers	4	5
	pi	4	6
	reciprocals	4	6
patterns	sound patterns	1	8
	decreasing patterns	1	9
	increasing patterns	1	9

CATEGORY	CONCEPT	LEVEL	MASTER
	pattern extension	1	9
	numeric patterns	1	10
	patterns (gen chars)	1	10
	shape patterns	1	10
	paths	2	4
	shrinking patterns	2	4
	geometric patterns extension	2	5
	pattern addition	2	5
	pattern subtraction	2	5
	growing patterns	2	6
	linear patterns	2	6
	variability	2	6
	geometric patterns	2	10
	repeating patterns	2	10
	pattern division	3	4
	pattern multiplication	3	4
	networks	3	7
perimeter &	circumference	2	10
circumference	perimeter	2	10
	circumference formula	3	5

CATEGORY	CONCEPT	LEVEL	MASTER
	perimeter formula	3	5
	radius	4	8
polynomials	add polynomials	4	0
	divide polynomials	4	0
	monomial	4	0
	multiply polynomials	4	0
	poly solution by bisection	4	0
	poly sol by successive approx	4	0
	poly solution by sign change	4	0
	polynomial (gen chars)	4	0
	subtract polynomials	4	0
precision & accuracy	absolute error	4	1
	relative error	4	1
probability	outcome	1	9
	certainty (gen chars)	1	10
	chance (gen chars)	1	10
	prediction (gen chars)	1	10
	sets	1	10
	fair chance	2	8
	constancy	2	10

CATEGORY	CONCEPT	LEVEL	MASTER
	event likelihood	2	10
	improbability	2	10
	probability (gen chars)	2	10
	counting procedure	3	1
	theoretical probability	3	1
	complementary events	3	4
	experimental model of prob	3	4
	tables depicting prob	3	4
	tree diagram model	3	4
	random variable	3	7
	mutually exclusive events	3	8
	certainty of conclusions	3	9
	area model	3	10
	odds	3	10
	random number	3	10
	addition counting procedure	4	0
	critical paths method	4	0
	discrete prob distribution	4	0
	discrete prob structures	4	0
	expected value	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	experimental prob	4	0
	factorial notation	4	0
	combination	4	1
	compound events	4	1
	conditional probability	4	1
	continuous prob distribution	4	1
	dependent events	4	1
	factorial	4	1
	independent events	4	1
	independent trials	4	1
	law of probability	4	1
	Monte Carlo simulation	4	2
	permutation	4	5
	Venn diagram model	4	5
proof	proof (general strategies)	2	10
	proof (multiple strategies)	3	4
	proof paragraph	4	0
	smallest set of rules	4	0
	theorem direct proof	4	0
	theorem indirect proof	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
	truth table proof	4	0
	validity	4	2
	theorem	4	5
problem solving strategies (ps)	estimate answers	1	10
	general ps process	1	10
	guess and check	1	10
	lists	1	10
	models	1	10
	tables	1	10
	equivalent representation	2	9
	diagrams	2	10
	process of elimination	2	10
	relevant vs irrelevant info	2	10
	restate problem	2	10
	symbolic representation	2	10
	trial & error	2	10
	algebraic representation	3	1
	problem space	3	4
	problem types	3	7
	complex problems	3	10

CATEGORY	CONCEPT	LEVEL	MASTER
	generate multiple ps strategies	3	10
	graphic representation	3	10
	method selection	3	10
	nonroutine vs. routine problems	3	10
	pattern recognition	3	10
	pictorial representation	3	10
	problem formulation	3	10
	simplification	3	10
	solution algorithms	3	10
	solution probabilities	3	10
	verbal representation	3	10
	work backwards	3	10
	written representation	3	10
	strategy generation techniques	4	6
	evaluate progress	4	8
	strategy efficiency	4	8
Pythagorean theorem	Pythagorean theorem	4	10
rate & velocity	constant rate of change	3	7
	distance formula	3	10
	growth rate	3	10

CATEGORY	CONCEPT	LEVEL	MASTER
	rate	3	10
	rate of change	3	10
	acceleration	4	10
	compound interest	4	10
	force	4	10
	interest (gen chars)	4	10
	speed	4	10
	velocity	4	10
ratio proportion	percent (gen chars)	2	10
percent	equal ratios	3	7
	proportional gain	3	8
	percent above 100	3	10
	percent below 1	3	10
	proportion (gen chars)	3	10
	proportional reasoning	3	10
	similar proportions	3	10
regression &	regression coefficient	4	0
correlation	spurious correlation	4	0
	regression line	4	1
	regression (gen chars)	4	2

CATEGORY	CONCEPT	LEVEL	MASTER
	correlation (gen chars)	4	10
scale	relative distance	2	10
	relative size	2	10
	scale (gen chars)	2	10
	scale maps	2	10
	blueprint	3	10
	scale drawings	3	10
	scale transformation	3	10
sequences & series	iterative sequence	3	0
	linear arithmetic sequence	3	0
	linear geometric sequence	3	0
	recursive sequence	3	0
	sequence (gen chars)	3	5
	recurrence equations	4	0
	recurrence relationships	4	0
	Fibonacci sequence	4	1
	series circuit	4	1
	series (gen char)	4	2
similarity & congruence	axis of symmetry	2	5
	congruence (gen chars)	2	5

CATEGORY	CONCEPT	LEVEL	MASTER
	shape similarity	2	10
	shape symmetry	2	10
	similarity (gen chars)	2	10
	similarity vs. congruence	3	4
	similar figures	4	1
statistics	confidence intervals	4	0
	parameters	4	0
	parameter estimates	4	0
	sample statistics	4	0
	summary statistics	4	1
	statistics (gen chars)	4	6
subtraction	difference	1	10
	subtraction facts	1	10
	subtract whole numbers	1	10
	associative prop subtract	2	5
	commutative property subtract	2	5
	mental subtraction	2	10
	subtraction algorithm	2	10
temperature	temp estimation	1	9
	temp (gen chars)	1	10

CATEGORY	CONCEPT	LEVEL	MASTER
	temp measurement	1	10
time	calendar	1	10
	days	1	10
	hours	1	10
	minutes	1	10
	seconds	1	10
	standard time measures	1	10
	time interval	1	10
	week	1	10
	year	1	10
	clock	2	10
	elapsed time	2	10
	time zones	2	10
trigonometry	cosine	4	0
	point of tangency	4	0
	right triangle geometry	4	0
	sine	4	0
	tangent	4	0
	trig relations	4	0
	trigonometric ratio	4	0

CATEGORY	CONCEPT	LEVEL	MASTER
units	same vs. different size units	1	10
	unit conversion	2	8
	unit differences	2	8
	standard vs. nonstandard units	2	10
	cubic units	3	4
	linear units	3	4
	square units	3	4
	unit size	3	7
	unit analysis	4	1
volume mass capacity	capacity (gen chars)	1	9
	volume (gen chars)	1	10
	volume rectangular solids	2	5
	mass	2	10
	volume measurement	2	10
	volume of irregular shapes	2	10
	volume of prisms	3	0
	volume of pyramids	3	0
	volume formula	3	2
	volume cylinders	3	8
	density	4	10

CATEGORY	CONCEPT	LEVEL	MASTER
weight	standard measure weight	1	10
	weight (gen chars)	1	10
	grams	2	10
	pounds	2	10

Level: 1 = K–2; 2 = 3–5; 3 = 6–8; 4 = 9–12
Master: Number of mathematics educators out of 10 who identified the concept as essential for all students to learn.

Footnotes

[1]Many systems for proposition analysis would decompose this argument into a subordinate proposition (see Turner & Greene, 1977). For the purposes of this study, arguments and objects included nominals with their modifying elements.

Table 1

Frequency of Instructional Concepts Across Four Grade Level Intervals

Level	Number of Instructional Concepts	Percent
Level 1 (K–12)	94	12.7
Level 2 (3–5)	216	29.1
Level 3 (6–8)	205	27.7
Level 4 (9–12)	226	31.5
Total	741	100.00

Table 2

Frequency of Instructional Concepts Within Categories Across Four Grade Level Intervals

Category	Level				Total
	1	2	3	4	
addition	4	4			8
area	1	6	3	3	13
central tendency & variability		4	5	5	14
charts & graphs	1	4	5	4	14
computation (general)	2	8	7	1	18
coordinate systems		6	3	2	11
data collect & samples	2	5	8	5	20
data distributions		4	4	10	18
decimals		6	3		9
direction position location	13				13
division		6	2		8
equations & inequalities		5	14	5	24
estimation	3	6	3	1	13
experiments		2	1	4	7

Category	Level				Total
	1	2	3	4	
exponents logs roots			8	14	22
expressions			4	7	11
factors multiples primes		5			5
figures & shapes	11	22	10	7	50
fractions	1	14	4	1	20
functions		1	13	31	45
length width height	2	6	3		11
lines & angles		12	4	7	23
math reasoning	2	11	4	8	25
matrices & vectors			1	13	14
measurement	1	6	6	11	24
metric system		4	2		6
money	2				2
motion geometry	4	4	7	4	19
multiplication		6	5		11
numbers & number systems	7	8	12	9	36

Category	Level				Total
	1	2	3	4	
patterns	7	10	3		20
perimeter circumference		2	2	1	5
polynomials				9	9
precision & accuracy				2	2
probability	5	5	12	19	41
proof		1	1	7	9
problem solving strategies	6	7	17	3	33
Pythagorean theorem				1	1
rate & velocity			5	6	11
ratio proportion percent		1	7		8
regression & correlation				5	5
scale		4	3		7
sequences & series			5	5	10
similarity & congruence		5	1	1	7
statistics				6	6
subtraction	3	4			7

Category	Level				Total
	1	2	3	4	
temperature	3				3
time	9	3			12
trigonometry				7	7
units	1	3	4	1	9
volume mass capacity	2	4	4	1	11
weight	2	2			4
Total	94	216	205	226	741

Table 3

Number of Instructional Concepts Considered Essential for All Students to Learn

Number of Mathematics Educators Identifying Concepts as Essential	Number of Concepts	Percent	Number of Concepts Greater Than or Equal To
0	143	19.3	741
1	41	5.5	598
2	23	3.1	557
3	8	1.1	534
4	53	7.2	526
5	69	9.3	473
6	23	3.1	404
7	26	3.5	381
8	39	5.3	355
9	17	2.3	316
10	299	40.4	299
Total	741	100.0	741