This document provides a summary of Recommendation 4 from the WWC practice guide *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **K–8**

LEVEL OF EVIDENCE: **Strong**

Recommendation

Interventions should include instruction on solving word problems that is based on common underlying structures.

Students who struggle with mathematics often face even larger difficulties in solving word problems. Thus, teachers should provide systematic, explicit instruction on solving word problems that is based in the problems' underlying structure. When students are taught the underlying structure of a word problem, they not only have greater success in problem-solving but can also gain insight into the deeper mathematical ideas in word problems. In this, teachers should make explicit connections between the structures of familiar and unfamiliar problems to help students learn when to apply a previously learned solution strategy.

How to carry out the recommendation

1. Teach students about the structure of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.

Instructional strategies from the examples

- Explicitly teach the important features of groups of problems with similar mathematical structures.
- Use visualizations to help students identify similarities in the structure of problems to determine appropriate solutions.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c, PS.7c
TEACHER: INST.PIC.2, INST.AM.4, INST.TCK.2

For groups of problems with similar mathematical structures, referred to as problem types, teachers should explicitly teach students the important underlying structural features that relate problems in the same problem type. Change problems (see Example 1) and compare problems (see Example 2) are two examples. Change problems always include a time element, and students use addition or subtraction to determine how much more or less. Compare problems focus on making comparisons between two different sets, and students need to calculate the unknown difference, unknown compared amount, or unknown referent amount.

To build understanding of each problem type, teachers should initially teach solution rules (i.e., guiding questions that lead to a solution equation) for each problem type through fully and partially worked examples, followed by student practice in pairs. Additionally, using visual representations can help students see the problem structure and determine an appropriate solution method.

Example 1. Sample problem types

Change Problems

The two problems here are addition and subtraction problems that students may be tempted to solve using an incorrect operation. In each case, students can draw a simple diagram like the one shown below, record the known quantities (two of three of A, B, and C), and then use the diagram to decide whether addition or subtraction is the correct operation to use to determine the unknown quantity.

Problem 1. Brad has a bottlecap collection. After Madhavi gave Brad 28 more bottlecaps, Brad had 111 bottlecaps. How many bottlecaps did Brad have before Madhavi gave him more?

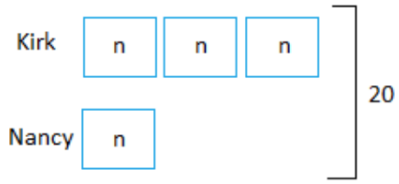
Problem 2. Brad has a bottlecap collection. After Brad gave 28 of his bottlecaps to Madhavi, he had 83 bottlecaps left. How many bottlecaps did Brad have before he gave Madhavi some?

Interventions should include instruction on solving word problems that is based on common underlying structures.

Compare Problems

Problem. Kirk has 3 times as many baseball cards as Nancy. Together, they have 20 baseball cards. How many cards does Kirk have?

Visual representation.



Note. Adapted from Example 1 on page 27 and Example 2 on page 28 of the practice guide.

2. Teach students to recognize the common underlying structure between familiar and unfamiliar problems and to transfer known solution methods from familiar to unfamiliar problems.

Instructional strategies from the examples

- Explicitly show students that not all pieces of information in the problem may be important for identifying the underlying structure.
- Provide opportunities for students to explain and discuss why a piece of information is relevant or irrelevant.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c, PS.4b, PS.7c
TEACHERS: INST.MS.2, INST.PIC.2, INST.AM.4, INST.TCK.2, INST.TH.2, PLAN.SW.1, PLAN.SW.3

Superficial changes (e.g., format, key vocabulary, the inclusion of irrelevant information) can often lead students to see a familiar problem as one that is new and unfamiliar. Format changes might be something like presenting a problem as an advertisement in a brochure rather than in a traditional paragraph form. Changes in key vocabulary might be identifying a fraction as half, one half, or $\frac{1}{2}$. These superficial changes are irrelevant to solving the problem but often lead to students struggling with identifying common underlying structures between new and old problems.

To facilitate transfer of methods to new problems from old problems, teachers should explicitly show students that not all pieces of information are relevant to identifying the underlying structure. Teachers should also provide opportunities for students to explain and discuss why a certain piece of information is relevant or irrelevant.

Example 2. Different problems with the same strategy

Problem 1. Mike wants to buy 1 pencil for each of his friends. Each packet of pencils contains 12 pencils. How many packets does Mike have to buy to give 1 pencil to each of his 13 friends?

Problem 2. Mike wants to buy 1 pencil for each of his friends. Sally wants to buy 10 pencils. Each box of pencils contains 12 pencils. How many boxes does Mike have to buy to give 1 pencil to each of his 13 friends?

Discussion. The structure of these two problems is identical, as is the required solution. However, the inclusion of irrelevant information in the second problem (i.e., Sally wants to buy 10 pencils) may cause students to see these two problems as different. Teachers should provide an opportunity for students to discuss what information in the problem is relevant to finding a solution or, if the students struggle to identify what is relevant and irrelevant, explicitly show students the relevant information.

Note. Adapted from Example 3 on page 39 of the practice guide.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
The curricular material may not classify problems into problem types.	The key issue is determining the problem types and an instructional sequence for teaching them so that students understand a set of problem structures and the related mathematics. To accomplish this, teachers may need assistance from a mathematics coach, a mathematics specialist, or a district or state curriculum guide.
As problems get complex, so will the problem types and the task of discriminating among them.	As problems become more complex (e.g., multistep problems), teachers may need to explicitly and systematically teach students how to differentiate one problem type from another. Again, teachers themselves may need additional support in determining problem types, justifying their responses, and explaining and modeling problem types to students.

Reference: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/2