This document provides a summary of Recommendation 5 from the WWC practice guide *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **K–8**

LEVEL OF EVIDENCE: **Moderate**

Recommendation

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Many students struggle with mathematics as a result of not having a strong understanding of the relationships between the abstract symbols used in mathematics and various visual representations. Helping students develop their ability to express mathematical ideas using visual representations, then correctly convert this into symbols is critical. As such, teachers should systematically teach students how to both develop visual representations and translate these into standard mathematical symbols they will use in solving the problem.

How to carry out the recommendation

1. Use visual representations such as number lines, arrays, and strip diagrams.

Instructional strategies from the examples

- Use visual representations (e.g., number lines, diagrams, pictorial representations) extensively and consistently.
- Explicitly link visual representations with standard mathematical symbols.

South Carolina standards alignment

MATHEMATICS: PS.2a, PS.2b, PS.4a
TEACHERS: INST.PIC.2, INST.TCK.2

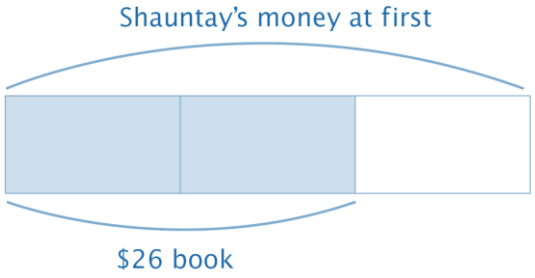
In early grades, teachers can use number lines, number paths, and other pictorial representations to help students develop foundational skills and procedural operations for counting, addition, and subtraction. In upper grades, diagrams and pictorial representations can be used to teach fractions or help students make sense of the underlying mathematical structure in word problems.

Example 1. Using strip diagrams to make sense of fractions

Problems
Shauntay spent $^{2}/_{3}$ of the money she had on a book that cost $26. How much money did Shauntay have before she bought the book?

Solution

Strip diagrams (also called model diagrams and bar diagrams) are one type of diagram that can be used to represent and help solve this problem. The full rectangle (consisting of all three equal parts joined together) represents Shauntay's money before she bought the book. Since she spent $^2/_3$ of her money on the book, two of the three equal parts represent the $26 she spent on the book.



Two parts of the strip diagram represent the $26 spent on the book. That means that one part would equal $13 ($\$26 \div 2 = \$13$). To find the amount she had at first, we would multiply $13 by 3. So Shauntay had $39 before she bought the book.

Note. Adapted from Example 6 on page 34 of the practice guide.

2. If visuals are not sufficient for developing accurate abstract thought and answers, use concrete manipulatives first. Although this can also be done with students in upper elementary and middle school grades, use of manipulatives with older students should be expeditious because the goal is to move toward understanding of—and facility with—visual representations, and finally, to the abstract.

Instructional strategies from the examples

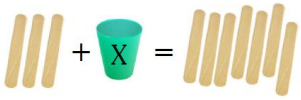
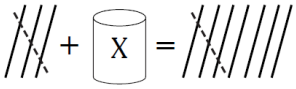
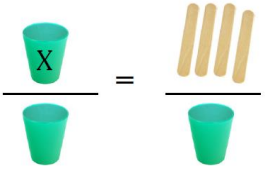
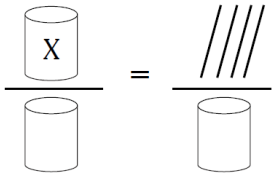
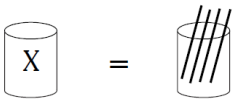
- Use manipulatives, when necessary, and focus on systematically phasing them out.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.2a, PS.2b, PS.4a
TEACHERS: INST.PIC.2, INST.AM.4, INST.TCK.2, PLAN.SW.3

Teachers should use concrete objects when using visual representations doesn't sufficiently support students understanding the more abstract symbols in math. However, teachers should focus on systematically fading the use of manipulatives to guide students toward reaching the more abstract level of using mathematical symbols. This involves explicitly teaching the concepts and operations with the concrete manipulatives and consistently connecting the visual with the abstract levels. Teachers should use consistent language across the different representations (concrete manipulatives, visual representation, and abstract symbols) to solidify connections for students.

Example 2. A set of matched concrete, visual, and abstract representations to teach solving single-variable equations

Solving the Equation with Concrete Manipulatives (Cups and Sticks)	Solving the Equation with Visual Representations of Cups and Sticks	Solving the Equation with Abstract Symbols
A 		$3 + 1x = 7$
B 		$-3 \quad -3$
C  D		$\frac{1x}{1} = \frac{4}{1}$
E 		$x = 4$

Note. Taken from Example 8 on page 35 of the practice guide.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
Many intervention materials provide very few examples of the use of visual representations.	Because many curricular materials do not include sufficient examples of visual representations, the interventionist may need the help of the mathematics coach or other teachers in developing the visuals. District staff can also arrange for the development of these materials for use throughout the district.
Some teachers or interventionists believe that instruction in concrete manipulatives requires too much time.	Expeditious use of manipulatives cannot be overemphasized. Since tiered interventions often rely on foundational concepts and procedures, the use of instruction at the concrete level allows for reinforcing and making explicit the foundational concepts and operations. Note that overemphasis on manipulatives can be counterproductive because students manipulating only concrete objects may not be learning to do math at an abstract level. The interventionist should use manipulatives in the initial stages strategically and then scaffold instruction to the abstract level. So, although it takes time to use manipulatives, this is not a major concern because concrete instruction will happen only rarely and expeditiously.
Some interventionists may not fully understand the mathematical ideas that underlie some of the representations. This is likely to be particularly true for topics involving negative numbers, proportional reasoning, and interpretations of fractions.	It is perfectly reasonable for districts to work with a local university faculty member, high school mathematics instructor, or mathematics specialist to provide relevant mathematics instruction to interventionists so they feel comfortable with the concepts. This can be coupled with professional development that addresses ways to explain these concepts in terms their students will understand.

Reference: Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/2