This document provides a summary of Recommendation 1 from the WWC practice guide *Developing Effective Fractions Instruction for Kindergarten through 8th Grade*. Full reference at the bottom of the last page.

CONTENT: **Mathematics**
GRADE LEVEL(S): **K–2**
LEVEL OF EVIDENCE: **Minimal**

Recommendation

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

Students develop a basic understanding of fractions before starting kindergarten through activities such as equally sharing a set of objects with a group of people and proportional reasoning. Teachers can build on this knowledge as they introduce students to the more formal concept of fractions (either as division or ratios), building on what students already understand about sharing equally. Although fraction concepts are usually not introduced until first or second grade, sharing activities can begin as early as pre-K, setting the stage for a deeper understanding of fractions.

Equal sharing	By age 4, students can distribute equal numbers of equal-size objects among a small number of recipients, and the ability to equally share improves with age. Sharing a set of discrete objects (e.g., 12 grapes shared among three students) tends to be easier for young students than sharing a single object (e.g., a candy bar), but by age 5 or 6, students are reasonably skilled at both.
Proportional relations	By age 6, students can match equivalent proportions represented by different geometric figures and by everyday objects of different shapes. One-half is an important landmark in comparing proportions; students more often succeed with comparisons in which one proportion is more than half and the other is less than half than on comparisons in which both proportions are more than half, or both are less than half (e.g., comparing $^1/_3$ to $^3/_5$ is easier than comparing $^2/_3$ to $^4/_5$). In addition, students can complete analogies based on

	proportional relations—for example, a half-circle is to a half-rectangle as a quarter-circle is to a quarter-rectangle.

Note. Taken from page 13 of the practice guide.

How to carry out the recommendation

1. Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.

Instructional strategies from the examples

- Begin equal-sharing activities with a set of objects that can be evenly distributed between two people, then move to larger numbers of people.
- Once students have gained facility with equally sharing a group of objects, move to partitioning a single object into fractional parts for equal sharing.
- As teachers increase the number of people to share with, teachers should consider using multiples of two to allow students to use repeated halving to help solve problems. Later, teachers should move to activities that can't be solved with repeated halving.

South Carolina standards alignment

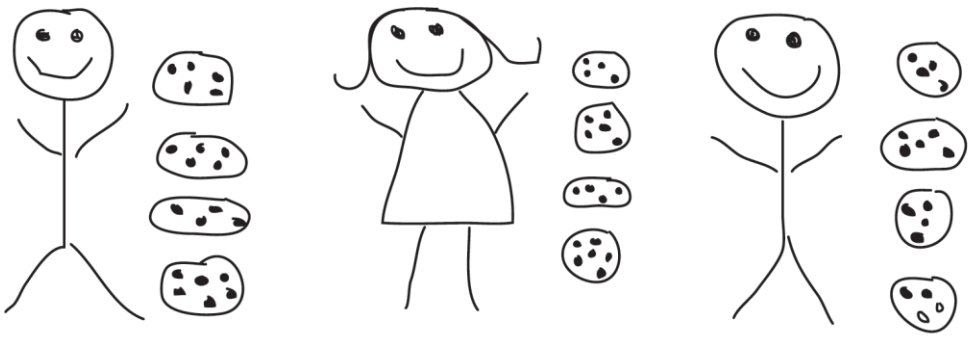
MATHEMATICS: 2.ATO.3
TEACHERS: INST.MS.2, PLAN.SW.3

Teachers should provide students with sharing activities that progressively build on their existing strategies for dividing, beginning with those involving equal sharing of a set of objects (first between two people, then increasing the number of people), then progressing to activities involving partitioning a set of objects, then a single object, into different fractional parts. Teachers should encourage students to use manipulatives (e.g., counters, beans) or drawings when doing these activities. As students work through activities and different representations, teachers can introduce formal fraction names and have students label fractions on their drawings. To most effectively build students' understanding, engage them in a variety of naming and labeling activities.

Sharing a set of objects. In early equal-sharing activities, teachers should have students solve problems that involve two people (then increase the number of

people) and a set of objects that can be divided equally with none left over. To further develop students' understanding, teachers should present situations by describing the number of items to be shared and the number of people who will be sharing, then have students work to determine the number of items each person should receive. As students gain success, teachers can pose a similar problem with the same number of items but increase the number of people. Note that problems of this type should always focus on equal sharing among the recipients.

Example of sharing a set of objects evenly among recipients

Problem
Three students want to share 12 cookies so that each student receives the same number of cookies. How many cookies should each student get?
Example of student solution strategy
Students can solve this problem by drawing three figures to represent the students and then drawing cookies by each figure, giving one cookie to the first student, one to the second, and one to the third, continuing until they have distributed 12 cookies to the three students, and then counting the number of cookies distributed to each student. Other students may solve the problem by simply dealing the cookies into three piles as if they were dealing cards. 

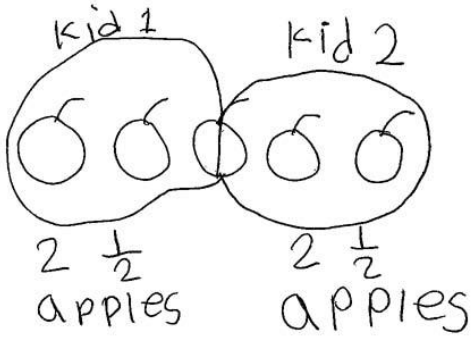
Note. Taken from Figure 1 on page 14 of the practice guide.

Partitioning a single object. After students gain facility with equal-sharing activities, teachers should present problems that involve dividing one or more objects into equal parts. Thus, the focus of the problems shifts from *how many* objects each person has, to *how much* of an object each person should get. For initial problems, teachers should present one object to be shared between people, then progress to multiple objects being divided into smaller parts to be shared equally among people.

Problems involving people equally dividing one object result in unit fractions (e.g., $^1/_2$, $^1/_3$, $^1/_5$), while the more complex problems of multiple objects and multiple people often result in non-unit fractions (e.g., $^1/_2$).

Teachers should also consider sharing among groups that are multiples of 2 (e.g., 2, 4, 8, 16 . . .). This allows students to partition using repeated halving strategies. Later, teachers should introduce problems that cannot be solved with this strategy to help students develop other strategies. For example, teachers may provide students with toothpicks or wooden sticks to lay across objects to represent cutting or partitioning.

Example of partitioning both multiple and single objects

Problem
Two students want to share five apples that are the same size so that both have the same amount to eat. Draw a picture to show what each student should receive.
Example of student solution strategy
Students might solve this problem by drawing five circles to represent the five apples and two figures to represent the two students. Students might then draw lines connecting each student to two apples. Finally, they might draw a line partitioning the final apple into two approximately equal parts and draw a line from each part to the two students. Alternatively, as in the picture below, students might draw a large circle representing each student, two apples within each circle, and a fifth apple straddling the circles representing the two students. In yet another possibility, students might divide each apple into two parts and then connect five half apples to the representation of each figure. 

Note. Taken from Figure 2 on page 15 of the practice guide.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

2. Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.

Instructional strategies from the examples

- As teachers gradually increase the number of people to share with, guide students to understand the inverse relationship between the number of people and the amount each person receives.
- Provide activities for students that also involve partitioning the number of sharers (e.g., move from one group sharing to the group now being divided equally at different tables and sharing).

South Carolina standards alignment

MATHEMATICS: 3.NSF.1a, 3.NSF.2a, 3.NSF.2b
TEACHERS: PLAN.SW.3

Building on the activities in Step 1 above, teachers can extend students' understanding to ordering fractions and identifying equivalent fractions using similar story problems involving a group of people sharing objects when students continue to use manipulatives and/or drawings to help make sense of the problem. However, teachers extend the scenarios to require fraction comparisons or identification of equivalent fractions by focusing on different aspects of students' solution strategies.

To help students better understand relative size, for example, teachers can gradually increase the number of people who will be sharing and have students compare the relative amounts that each person receives. In this, teachers should guide students to observe that increasing the number of people reduces the amount that each person receives and vice versa. Teachers should help students link this idea to the formal fraction names they have been using to identify the quantities, guiding them to use these names to discuss the results of their solutions (e.g., $^1/_3$ of an object is greater than $^1/_4$ of that same object).

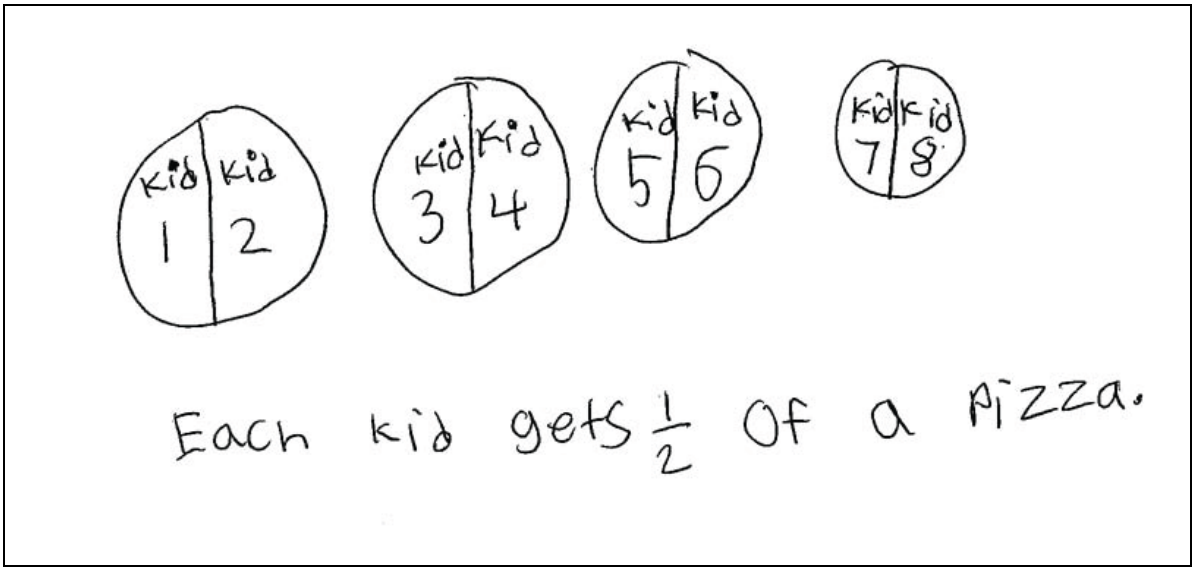
In these scenarios, teachers should include both of the following approaches:

- **Partition objects into larger and smaller pieces.** Teachers should guide students to think about different ways to solve the same problem. This might involve partitioning the object(s) into smaller or larger pieces while ensuring that each person receives an equal amount. In this way, teachers can help students see that groups of the smaller pieces can be combined to be the same size as a larger piece. For example, as shown below, a student may solve

the problem of eight people sharing four pizzas by cutting each pizza in half and giving each person half of a pizza. This problem could also be solved by cutting each pizza into quarters and giving each person two quarters. Thus, one half is the same as, or equivalent to, two quarters (i.e., $^1/_2 = {}^2/_4$).

- **Partition the number of sharers and the number of items.** In this approach, teachers help students build the idea of fraction equivalence by partitioning both the number of sharers and the number of objects. For example, in the problem shared previously, a student may partition each pizza into eighths and give each person four pieces or four eighths. The teacher could extend the problem to have the student compare what would happen if the people were split into two tables so that each table of four had to share two pizzas . . . or that people were split into four tables. In each case, each person still receives one-half of the pizza. Every situation is equivalent.

Example of student work for sharing four pizzas among eight students



Note. Taken from Figure 3 on page 16 of the practice guide.

Teachers can also help students develop their understanding of equivalent fractions by using "missing value" problems. Here, students need to determine the number of objects they would need to result in an equivalent share. For example, the practice guide (p. 17) presents the following: "If six students share eight oranges at one table, how many oranges are needed at a table of three students to ensure each student receives the same amount?" (Note: Further scenarios and examples of students' thinking are provided on p. 17 of the practice guide.)

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

3. Build on students' informal understanding to develop a more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Instructional strategies from the examples

- Use proportional relations, covariation, and/or patterns to help develop students' proportional thinking.
- Use scenarios that can't directly be quantified to help students think about proportional relations outside of a full focus on numbers.

South Carolina standards alignment

MATHEMATICS: 3.NSF
TEACHERS: PLAN.SW.3

To help build students' understanding of proportional reasoning, teachers might use a story like *Goldilocks and the Three Bears* that asks students to think about the proportional relationships between pairs of objects without providing specific numbers (e.g., Papa Bear goes with the large chair, Baby Bear goes with the small bed). Here are some examples of different relations teachers might use:

- **Proportional relations.** As with *Goldilocks and the Three Bears*, teachers present students with basic proportional relations that are not quantified (e.g., do not present actual values). For example, how many students would it take to balance a seesaw with one, two, or three adults on the other side?
- **Covariations.** These types of scenarios involve one quantity increasing while another is also increasing (e.g., the age of a student and the height of the student).
- **Patterns.** Teachers can use simple repeating patterns to develop a scenario (e.g., RRYRRYRRYRRY), then discuss with students how many red beads there are for every yellow bead. Students can then change the pattern to a different ratio or extend the pattern to solidify proportional reasoning.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
Students are unable to draw equal-sized parts.	Teachers should let students know that it's okay to draw parts that aren't exactly equal as long as they recognize that the parts are supposed to be equal when they work on solving the problem.
Students do not share all the items (non-exhaustive sharing) or do not create equal shares.	Although teachers are building on students' intuitive understanding of sharing equally, students will still make mistakes, such as not sharing all the objects, especially when the problem involves partitioning one or more of the objects. Teachers should focus on helping students understand that sharing involves using all the objects (e.g., letting students know that each person needs to get all they possibly can). If students do not create equal shares, teachers can reframe by reminding students that each person needs to receive the same amount (e.g., things need to be shared fairly). Equal sharing like this helps lay the foundation for students' understanding of equivalent fractions and equivalent magnitude difference (e.g., that a difference of ½ is the same whether it's between 0 and ½ or 23 and 23 ½).
When creating equal shares, students do not distinguish between the number of things shared and the quantity shared.	Limited experience often leads students to make the mistake of giving each person the same number of items rather than thinking about the amount, especially when items shared may be of different sizes. Teachers can use color cues to help students distinguish between the number and the amount. For example, if five small and five large food items are being shared between two dogs, the teacher may color the large items green and the small items red, give all five green items to one dog and all five red items to the other, then ask students if the dogs received the same amount of food.

Reference: Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade* (NCEE 2010-4039). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/15