This document provides a summary of Recommendation 2 from the WWC practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*. Full reference at the bottom of last page.

CONTENT: **Mathematics**
GRADE LEVEL(S): **K–8**
LEVEL OF EVIDENCE: **Moderate**

Recommendation

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Initial fraction learning often starts with the concept of fractions as part of a whole, but that understanding does not express the fact that fractions are numbers with magnitudes that can be compared (e.g., ordered, considered equivalent). Missing this understanding is often at the root of many misconceptions about fractions (e.g., adding to fractions by adding the numerators, then adding the denominators; not seeing fractions as numbers or units of measurement). Using number lines is an effective way to help develop an understanding of fractions as numbers with magnitudes because they can provide a clear picture of the magnitude of fractions, the relationship between fractions and whole numbers, and the relationship between fractions, decimals, and percentages. Number lines also provide a foundation for students' number sense with fractions and a way to visualize negative fractions.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

How to carry out the recommendation

1. Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.

Instructional strategies from the examples

- Use measurement activities to develop the idea that fractions allow for more precise measurement than whole numbers.
- Present situations in which fractions are used to solve problems that cannot be solved only using whole numbers.
- Show students the various measurement lines on a measuring cup and convey the importance of fractions in describing quantities.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.2a, PS.2b
TEACHERS: INST.PIC.2, INST.TCK.2, PLAN.SW.3

Helping students view fractions as numbers opens the door for them to use fractions to measure quantities and understand that using fractions allows for more precise measurement of objects. Fractions can also help solve certain problems that cannot be solved using only whole numbers (e.g., describing the amount of sugar needed for a recipe that asks for more than 1 cup but less than 2 cups). Fraction strips are length models that teachers can use to reinforce the idea of fractions representing quantities. These fraction strips are also known as fraction strip drawings, strip diagrams, bar strip diagrams, and tape diagrams.

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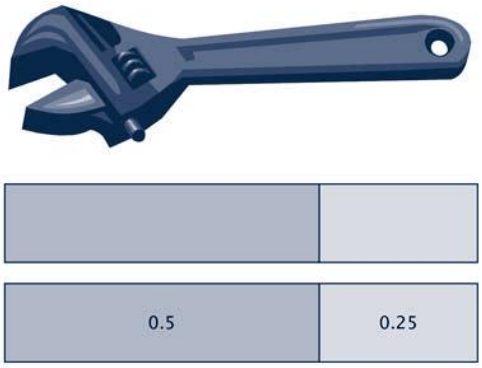
Example of measurement activities with fraction strips

Teachers can use fraction strips as the basis for measurement activities to reinforce the concept that fractions are numbers that represent quantities.

Using fraction strips to measure an object



To start, students can take a strip of card stock or construction paper that represents the initial unit of measure (i.e., a whole) and use that strip to measure objects in the classroom (desk, chalkboard, book, etc.). When the length of an object is not equal to a whole number of strips, teachers can provide students with strips that represent fractional amounts of the original strip. For example, a student might use three whole strips and a half strip to measure a desk.

Teachers should emphasize that fraction strips represent different units of measure and should have students measure the same object first using only whole strips and then using a fractional strip. Teachers should discuss how the length of the object remains the same but how different units of measure allow for better precision in describing it. Students should realize that the size of the subsequently presented fraction strip is defined by the size of the original strip (i.e., a half strip is equal to one half of the length of the original strip).

Note. Taken from Example 1 on page 21 in the practice guide.

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2. Provide opportunities for students to locate and compare fractions on number lines.

Instructional strategies from the examples

- Provide opportunities for students to locate and compare fractions on number lines, beginning with fully labeled number lines and gradually reducing the labels.
- Use number lines to compare various fractions to whole numbers greater than one.
- Encourage students to think about the distance between two fractions.

South Carolina standards alignment

MATHEMATICS: PS.2a, PS.2b
TEACHERS: INST.PIC.2

Activities involving comparing fractions should include a variety of fraction forms, such as proper fractions, improper fractions, mixed numbers, whole numbers, decimals, and percentages. Teachers should start by providing number lines with fractional parts already marked and move to numbers lines that are more minimally labeled. This helps avoid student errors in partitioning accurately and also allows for location and comparison of fractions whose locations are indicated (e.g., $^3/_8$ and $^5/_8$ on a ruler), as well as fractions whose denominators are a factor of the indicated fractions (e.g., $^1/_4$ and $^3/_4$) and fractions between those indicated (e.g., $^1/_7$ and $^3/_5$).

Activities should also involve comparison with whole numbers, include fractions equivalent to whole numbers (e.g., locating 1 and $^8/_8$) and comparing fractions of various sizes to whole numbers greater than one (e.g., locating $^{10}/_3$ on a number line, with 0 at the left end and 5 at the right end). To assist students in comparing fractions with different denominators, teachers can label number lines with one fractional-unit sequence above and a different sequence below.

One activity teachers can use with the whole class involves drawing a number line on the board and having students estimate and mark where different fractions fall. The teacher can guide students' discussion about fractions yet to be placed, focusing on maintaining the correct order and encouraging thinking about the distance between fractions. Again, including a variety of fraction types, and also decimals and percentages, helps solidify student understanding.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Example of introducing fractions on a number line

To illustrate the location of $^{3}/_{5}$ on a 0-to-5 number line, a teacher might first mark and label the location of 1 and then divide the space between each whole number into five equal-size parts. After this, they might add the labels $^{0}/_{5}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{3}/_{5}$, $^{4}/_{5}$, and $^{5}/_{5}$ in the 0–1 portion of the number line and highlight the location of $^{3}/_{5}$. Displaying whole numbers as fractions (e.g., $^{5}/_{5}$) allows teachers to discuss what it means to describe whole numbers in terms of fractions and to clarify that whole numbers are fractions, too.

Note. Adapted from Example 2 on page 22 in the practice guide.

3. Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.

Instructional strategies from the examples

- Use number lines to:
    - Illustrate that equivalent fractions describe the same magnitude.
    - Illustrate that an infinite number of fractions exist between any two other fractions.
    - Introduce negative fractions.
    - Convey the symmetry of positive and negative fractions about zero.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.2a, PS.2b
TEACHERS: INST.PIC.2, INST.TCK.2, PLAN.SW.3

Number lines are also useful tools for helping students think about and identify equivalent fractions, negative fractions, and fraction density.

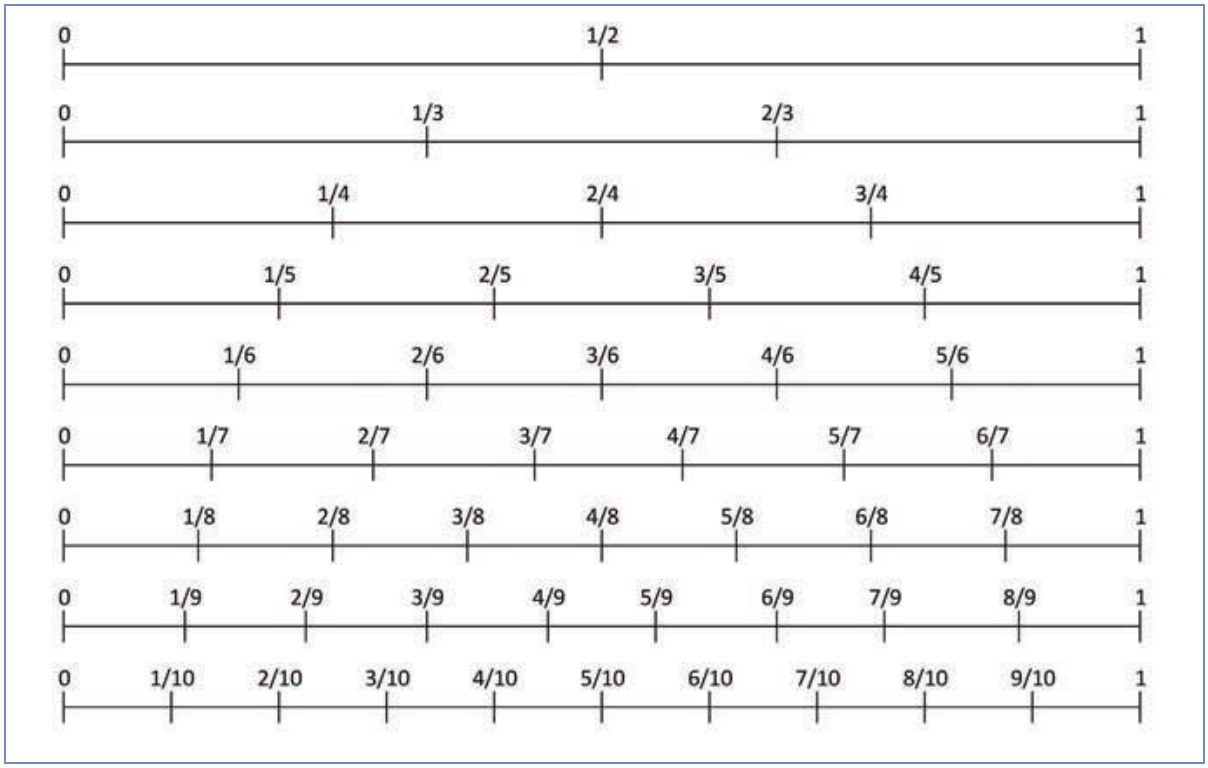
To illustrate that equivalent fractions have the same magnitude, a teacher might provide a number line labeled with one fractional sequence on top and a different fractional sequence on the bottom. Students can locate fractions on each of the sequences and discuss how they are equivalent when they are in the same location on the number line. Discussions about equivalent fractions should build the measurement activities from Step 1 above, using rulers and fraction strips to reinforce.

Another challenging concept is for students to understand that an infinite number of fractions exist between any two other fractions on the number line. This is an important difference between fractions and whole numbers. To help students understand this concept, teachers can ask them to successively partition a portion of the number line into smaller and smaller unit fractions (e.g., repeatedly dividing a whole number segment in half). Similarly, teachers should use decimals and percentages to represent this concept.

The number line provides an excellent visual representation of fractions both greater than and less than zero. By providing number lines that include marks and labels for

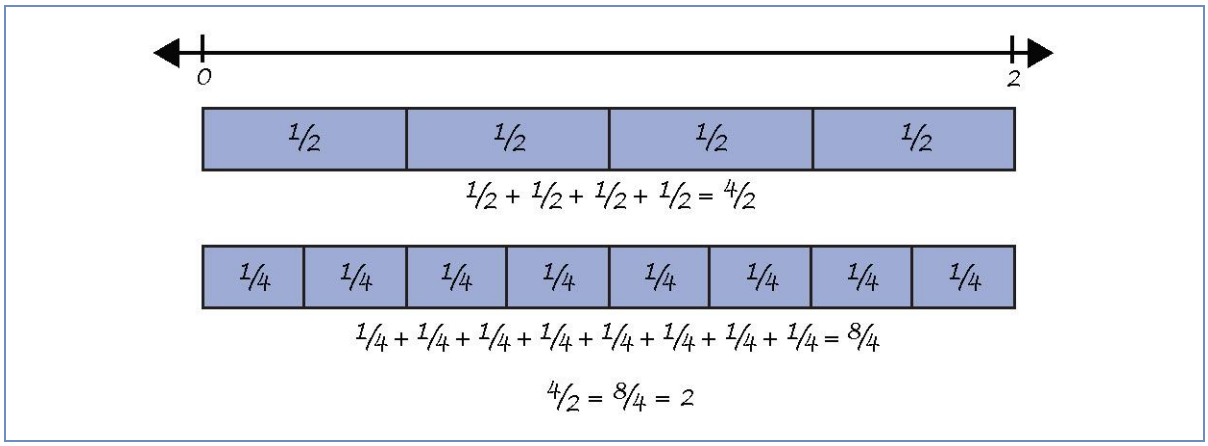
zero, several positive fractions, and several negative fractions with the same absolute values as the positive fractions, teachers can help convey the symmetry about zero of positive and negative fractions.

Example of using numbers lines to show equivalence of fractions



Note. Taken from Example 4 on page 23 in the practice guide.

Example of using fraction strips to demonstrate equivalent fractions



Note. Taken from Example 5 on page 24 in the practice guide.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

4. Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.

Instructional strategies from the examples

- Use a number line with common fractions listed above with equivalent decimals and percentages below.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.2a, PS.2b, PS.6b, 6.NS.1, 6.NS.9, 6.RP.3e, 7.NS.5, 8.NS.3
TEACHERS: INST.PIC.2, INST.AM.4, INST.TCK.2, PLAN.SW.3

Number lines can provide a useful tool in helping students develop a broad view of fractions as numbers, including understanding that fractions can be represented as decimals and percentages, as well as common fractions. Fractions, decimals, and percentages are just different ways of representing the same number. By using a number line with common fractions listed above and decimals or percentages below, students can locate and compare fractions, decimals, and percentages on the same number line.

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Potential roadblocks and how to address them

Roadblock	Suggested Approach
Students try to partition the number line into fourths by drawing four hash marks rather than three, or they treat the whole number line as the unit.	Teachers may need to teach students that "fourths" is represented as four equal segments between two whole numbers, demonstrating that three equally spaced hatch marks divides the space into four equal parts. Later, guide students to generalize this rule to know that to divide a portion of the number line between two whole numbers into $1/n$ units involves drawing $n - 1$ hatch marks between the two whole numbers.
When students locate fractions on the number line, they treat the numbers in the fraction as whole numbers (e.g., placing 3/4 between 3 and 4).	This error is based in students' misconception where they try to apply whole number understanding to fractions. Teachers can help address this misconception through using number lines and providing contrasting cases where students locate, for example, 3 and 4 on a 0-to-4 number line, then locate $3/4$ between 0 and 1. Teachers should follow this activity with discussion about why each is placed where it is on the number line.
Students have difficulty understanding that two equivalent fractions are the same point on a number line.	Teachers can show students one set of labels above the number line and another below the number line. For example, halves could be marked above and eighths below. Teachers can then point to the equivalent positions of $1/2$ and $4/8$ or 1, $2/2$, and $8/8$, and so on. Another approach would be for students to create a number line showing $1/2$ and another number line showing $4/8$, and then compare the two. Teachers can line up the two number lines and lead a discussion about equivalent fractions.

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Roadblock	Suggested Approach
The curriculum materials used by my school district focus on part-whole representations and do not use the number line as a key representational tool for fraction concepts and operations.	Understanding that fractions can represent part of a whole is only one use. Using measurement and number lines provides an additional context to help students build a rich understanding of fractions. Teachers can also use manipulatives designed to represent parts of a whole for measurement purposes; when doing this, teachers should make sure students aren't simply counting pieces represented by the numerator and denominator but are instead seeing the fraction as a single quantity. Using number lines that are unmarked between endpoints can help, because they do not provide objects to count. Additionally, seek out other textbooks and resources that may provide examples of using fractions as measures of quantity.

Reference: Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade* (NCEE 2010-4039). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/15