This document provides a summary of Recommendation 2 from the WWC practice guide *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students.* Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **6–12**

LEVEL OF EVIDENCE: **Minimal**

Recommendation

Teach students to utilize the structure of algebraic representations.

Examining the underlying structure of an algebra problem (the algebraic representation), regardless of how the problem itself is communicated (for example, symbolic, numeric, verbal, or graphic), can help students see similarities among problems, connections, and solution paths. It also leads to development of understanding about algebraic expressions.

How to carry out the recommendation

1. Promote the use of language that reflects mathematical structure.

Instructional strategies from the examples

- Phrase algebra solution steps in precise mathematical language to communicate the logical meaning of a problem's structure, operations, solution steps, and strategies.
- Use precise mathematical language to help students analyze and verbally describe the specific features that make up the structure of algebraic representations.
- Rephrase student solutions and responses using appropriate mathematical language.

South Carolina standards alignment

MATHEMATICS: PS.3a, PS.3b, PS.6c
TEACHERS: INST.PIC.3, INST.PIC.4, INST.TCK.2, PLAN.SW.2

Using precise language is important for helping students understand algebraic structure. Teachers should use and model precise mathematical language that is related to the structure of the algebraic expression. (See Example 2.2 on page 18 in the practice guide for an example using the distributive property.) Doing so will not only help students develop an understanding of the structure but also lay a foundation for them to reflect, ask questions, and create appropriate representations. Teachers should restate students' responses, using the appropriate mathematical language, to help them grow in their ability to use precise language. Guiding students to use more precise mathematical language helps them focus on and build understanding of the mathematical validity of a problem.

Examples of imprecise language with more precise restatements

Imprecise Language	Precise Mathematical Language
Take out the x.	Factor x from the expression. Divide both sides of the equation by x, with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.

Note. Taken from Example 2.3 on page 18 in the practice guide.

2. Encourage students to use reflective questioning to notice structure as they solve problems.

Instructional strategies from the examples

- Model reflective questioning to students by thinking aloud while solving a problem.
- Share a list of common questions students can ask themselves while s olving a problem.

South Carolina standards alignment

MATHEMATICS: PS.1c, PS.1d, PS.3a, PS.3d, PS.7b, PS.7c
TEACHERS: INST.MS.1, INST.MS.2, INST.AM.4, INST.AM.5, INST.TH.2, INST.PS.1, PLAN.SW.1, PLAN.Desc.1

When students ask themselves questions about solving a problem, they are more likely to think about the structure of the problem and solution methods they might use. Teachers should model reflective questions using think-alouds focused on algebraic structure when demonstrating problem-solving. Additionally, providing lists of reflective questions can be helpful as students move from modeling to more independent work. Teachers should encourage students to work in pairs to develop and record their own reflective questions and then carry this practice to independent work.

Examples of reflective questions

- What am I being asked to do in this problem?
- How would I describe this problem using precise mathematical language?
- Is this problem structured similarly to another problem I've seen before?
- How many variables are there?
- What am I trying to solve for?
- What are the relationships between the quantities in this expression or equation?
- How will the placement of the quantities and the operations impact what I do first?

Note. Taken from Example 2.5 on page 20 in the practice guide.

3. Teach students that different algebraic representations can convey different information about an algebra problem.

Instructional strategies from the examples

- Present equations in different forms and ask students to identify similarities and differences.
- Help students see that different representations based on the same information can display the information differently.
- Incorporate diagrams into instruction to demonstrate similarities and differences between representations of algebra problems.

South Carolina standards alignment

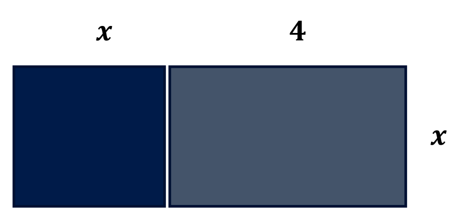
MATHEMATICS: PS.1c, PS.2b, PS.2d, PS.4b, PS.7b
TEACHERS: INST.PIC.2, INST.AM.4, INST.TCK.2

Teachers should encourage students to identify and explain different representations of the same problem in order to help them better understand the underlying mathematical structure. During whole-class instruction, teachers should provide a model of both the similarities and differences, showing how different representations of the same information might make solving problems easier. Teachers should also encourage students to see how some representations might better present information about the structure of the problem than others might. As needed, diagrams can help students visualize the problem structure, organize their thoughts about how to solve the problem, and transform the problem into another representation.

Example of using different representations to understand the structure of a problem

Word problem
Ray and Juan both have community garden plots. Ray has a rectangular plot. Juan has a square plot, and each side of his plot is x yards wide. Ray and Juan's plots share one full border; the length of Ray's plot on an unshared side is 4 yards. If Juan and Ray put in a fence around both of their plots, the area of the fenced space would be 21 square yards. How wide is the shared border? *The statement of the problem is one representation of a relationship among three quantities, which are the total area of 21 square yards, the area of Ray's plot, and the area of Juan's plot. Students typically move to other representations to solve the problem. They might draw a diagram and produce an equation, and then solve the equation algebraically or graphically.*

Diagram

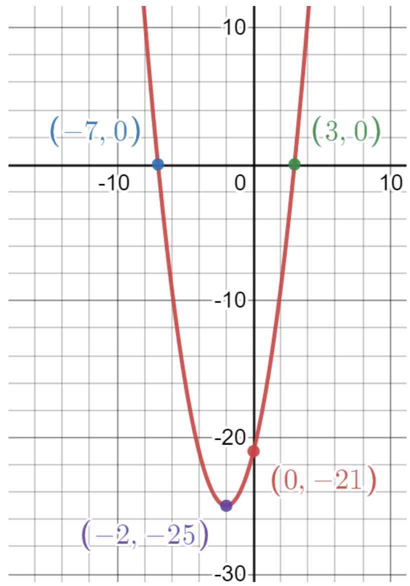


The diagram represents the two garden plots with a common border and a 4-yard unshared side of Ray's plot. The diagram also represents one large rectangle composed of two rectangles to illustrate that the total area is equal to the area of Ray's plot plus the area of Juan's plot. Using the rectangles, the given lengths, and the total area of 21 square yards, students can produce and solve an equation.

Students can use the diagram to see the structure of the problem as the equivalence of a total area to the sum of two parts and to express it as an equation. After solving the equation for x, students can explain why there are two possible solutions for the quadratic equation, and why -7 doesn't yield an answer to the question in the word problem.

Equation
Equation representing the equivalent areas in square yards: $\mathbf{21 = x(4 + x)}$ Equation in standard form: $\mathbf{0 = x^2 + 4x - 21}$ Equation in factored form: $\mathbf{0 = (x + 7)(x - 3)}$ Total area = 21 yd^2 Area = length × width Area = $\mathbf{x(4 + x)}$ $\mathbf{21 = x(4 + x)}$ $\mathbf{21 = 4x + x^2}$ $\mathbf{0 = x^2 + 4x - 21}$ $\mathbf{0 = (x + 7)(x - 3)}$ $\mathbf{x = -7 \; x = 3}$ *Students will likely come to the standard form first when solving this problem, then will need to factor to reach the possible solutions for x.* *Students should recognize that the quadratic expression can be factored. The values of x that make the factored expression on the right side of the equation equal to zero can be read from the structure of the expression as a product. For a product to be zero, one of the factors has to be zero, so x is -7 or 3.*

Graph



Students can find where an expression equals zero by thinking of the expression as a function, graphing it, and seeing where the graph crosses the x-axis.

The x-intercepts of the parabola can be read from the factored form. The y-intercept can be read from the standard form, and that form is helpful in determining the vertex of the parabola.

The graph is a parabola because it is a quadratic equation, and the direction in which the parabola opens depends on the sign of the coefficient of x^2.

Note. Adapted from Example 2.8 on pages 22–23 in the practice guide.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
I like to simplify mathematical language, and my students seem to respond positively to my use of simplified and informal language as well. Doesn't this approach make it easier for students than using complicated mathematical language?	Using precise mathematical language ensures that students understand the algebraic concepts in the problem. Language can be simplified but should still clearly link to mathematical structure and ideas. When students use informal language, teachers should restate in correct mathematical language to build students' capacity to use precise language.
My students race through problems. How do I get students to slow down, pause to ask themselves questions, and think about the problem?	Using more challenging or less familiar problems may slow students down and require them to pay attention and notice structure. Another possibility is to have students answer reflective questions as they work or solve problems using multiple solution paths or representations.
Diagrams don't seem to be very useful to some of my students.	Students may not need diagrams to solve problems, but diagrams can help them notice and understand structural components. Make this explicit to students, and continue to model the use of diagrams.

References: Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., & Zbiek, R. M. (2019). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2015-4010). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/20