This document provides a summary of Recommendation 3 from the WWC practice guide *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students.* Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **6–12**

LEVEL OF EVIDENCE: **Moderate**

Recommendation

Teach students to intentionally choose from alternative algebraic strategies when solving problems.

Students benefit from learning multiple algebraic strategies to bring to bear on problem-solving. Strategies are more general and abstract than memorized algorithms. Using strategies both requires students to have and provides them with more flexibility in solving problems. Students should not be expected to memorize all possible strategies but have access to multiple strategies for a given problem.

How to carry out the recommendation

1. Teach students to recognize and generate strategies for solving problems.

Instructional strategies from the examples

- Provide students with examples that illustrate the use of multiple algebraic strategies, including standard, commonly used strategies, as well as alternative strategies that may be less obvious.
- Use solved problems that demonstrate how the same problem could be solved with different strategies, as well as how different problems could be solved with the same strategy.
- After students find a solution to a problem, challenge them to solve the problem another way.

Teach students to intentionally choose from alternative algebraic strategies when solving problems.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c, PS.3a, PS.7b
TEACHERS: INST.MS.2, INST.PIC.2, INST.AM.4, INST.AM.9, INST.TCK.2, INST.TH.2, INST.PS.1, PLAN.SW.1, PLAN.Desc.1

Teachers can provide well-known, as well as lesser-known, strategies for approaching algebraic problems so students can observe which are effective and efficient in various cases. Teachers should provide solved problems to demonstrate multiple strategies for a single problem as well as strategies that are effective across multiple problems. Doing so will reinforce flexibility of strategy use. In both whole-class instruction and partner work, students should discuss and communicate why a particular strategy is useful.

Examples using different solution strategies

<table>
<tr><th>Conventional Solution Method</th><th colspan="2">Alternative Solution Method(s)</th></tr>
<tr><td colspan="3">Evaluate $\mathbf{2a + 4b - 7a + 2b - 8a}$ if $\mathbf{a = 1}$ and $\mathbf{b = 7}$.</td></tr>
<tr><td>$\mathbf{2a + 4b - 7a + 2b - 8a}$
$\mathbf{2(1) + 4(7) - 7(1) + 2(7) - 8(1)}$
$\mathbf{2 + 28 - 7 + 14 - 8}$
$\mathbf{29}$</td><td colspan="2">$2a + 4b - 7a + 2b - 8a$
$-13a + 6b$
$-13(1) + 6(7)$
$-13 + 42$
29</td></tr>
<tr><td colspan="3">Our restaurant bill, including tax but before tip, was $16.00. If we wanted to leave exactly 15% tip, how much money should we leave in total?</td></tr>
<tr><td>$\mathbf{16.00 * 1.15 = x}$

$\mathbf{x = \$18.40}$</td><td colspan="2">10% of $16.00 is $1.60, and half of $1.60 is $0.80, which totals $2.40, so the total bill with tip would be $16.00 + $2.40 or $18.40.</td></tr>
<tr><td colspan="3">Solve for $\mathbf{x}$: $\mathbf{3(x + 1) = 15}$</td></tr>
<tr><td>$\mathbf{3(x + 1) = 15}$
$\mathbf{3x + 3 = 15}$
$\mathbf{3x = 12}$
$\mathbf{x = 4}$</td><td>$3(x + 1) = 15$
$x + 1 = 5$
$x = 4$</td><td>I know that $3 \times 5 = 15$, so $x + 1$ has to equal 5. That means $x = 4$.</td></tr>
</table>

Note. Adapted from Example 3.1 on page 28 in the practice.

Examples of two different solution strategies to solve the same problem

Strategy 1: Devon's Solution—Apply Distributive Property First	
Solution steps	Labeled steps
$10(y+2) = 6(y+2) + 16$	Distribute
$10y + 20 = 6y + 12 + 16$	Combine like terms
$10y + 20 = 6y + 28$	Subtract $6y$ from both sides
$4y + 20 = 28$	Subtract 20 from both sides
$4y = 8$	Divide by 4 on both sides
$y = 2$	
Strategy 2: Elena's Solution—Collect Like Terms First	
Solution steps	Labeled steps
$10(y+2) = 6(y+2) + 16$	Subtract $6(y = 2)$ on both sides
$4(y+2) = 16$	Divide by 4 on both sides
$y + 2 = 4$	Subtract 2 from both sides
$y = 2$	

Prompts to Accompany the Comparison of Problems, Strategies, and Solutions

- What similarities do you notice? What differences do you notice?
- To solve this problem, what did each person do first? Is that valid mathematically? Was that useful in this problem?
- What connections do you see between the two examples?
- How was Devon reasoning through the problem? How was Elena reasoning through the problem?
- What were they doing differently? How was their reasoning similar? Did they both get the correct solution?
- Will Devon's strategy always work? What about Elena's? Is there another reasonable strategy?
- Which strategy do you prefer? Why?

Note. Taken from Example 3.2 on page 29 in the practice guide.

Teach students to intentionally choose from alternative algebraic strategies when solving problems.

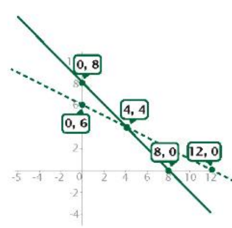
Teachers should introduce one or two strategies at a time to allow students to process new information. They should then work with students to determine which strategies are most effective and efficient through reflective questions that teachers provide or students develop themselves. Students should begin by examining solved problems and then discuss and select strategies for solving other problems during group and individual work. Finally, after students solve a problem, teachers can challenge them to use a different strategy to solve it.

Examples of reflective questions for selecting and considering solution strategies

- What strategies could I use to solve this problem? How many possible strategies are there?
- Of the strategies I know, which seem to best fit this particular problem? Why?
- Is there anything special about this problem that suggests that a particular strategy is or is not applicable or a good idea?
- Why did I choose this strategy to solve this problem?
- Could I use another strategy to check my answer? Is that strategy sufficiently different from the one I originally used?

Note. Taken from Example 3.4 on page 30 in the practice guide.

Examples of possible strategies for solving linear systems

Problem Statement	Solution Strategy	Solution Steps	Notes About Strategies
$5x + 10y = 60$ $x + y = 8$	Graph using *x*- and *y*-intercepts	$5x + 10y = 60$ $(12, 0)(0, 6)$ $x + y = 8$ $(8, 0)(0, 8)$ 	The *x*- and *y*-intercepts are integers and easy to find in these two equations, so graphing by hand to find the point of intersection might be a good strategy to use.

Problem Statement	Solution Strategy	Solution Steps	Notes About Strategies
$-2x + y = 7$ $x = 6y + 2$	Substitution	$-2x + y = 7$ $\underline{x = 6y + 2}$ $-2(6y + 2) + y = 7$ $-12y - 4 + y = 7$ $-11y = 11$ $y = -1$	Because one of the equations in this system is already written in the form of *x* =, it makes sense to use the substitution strategy.
$2x + y = 6$ $x - y = 9$	Elimination	$2x + y = 6$ $\underline{x - y = 9}$ $3x = 15$ $x = 5$ $2(5) + y = 6$ $y = -4$	Because the coefficients of the *y* terms are equal in absolute value but have opposite signs, the strategy of elimination may be a natural fit for this system.
$y = 100 + 4x$ $y = 25 + 7x$	Properties of equality	$y = 100 + 4x$ $\underline{y = 25 + 7x}$ $100 + 4x = 25 + 7x$ $75 = 3x$ $x = 25$	Since both equations are in the form of *y* =, it would be logical to set the two expressions in *x* equal to each other and solve for *x*.

Note. Adapted from Example 3.6 on page 32 in the practice guide.

2. Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy when solving problems.

Instructional strategies from the examples

- Have students describe their reasoning while analyzing the problem structure, determining their solution strategy, solving a problem, and/or analyzing another student's solution.
- When introducing group activities, model how to work with a partner to discuss potential strategies, how to label the steps of each strategy, and how to explain the similarities and differences observed between strategies.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.1b, PS.1c, PS.1d, PS.3a, PS.6c, PS.7c
TEACHERS: INST.AM.4, INST.AM.7, INST.TCK.2, INST.PS.1, PLAN.SW.1, PLAN.SW.2, PLAN.Desc.1

To help students better understand their choices and goals in problem-solving, teachers should provide multiple opportunities to analyze problem structures, determine solution strategies, describe reasoning while solving problems, and analyze other students' solution strategies. Students should demonstrate their strategic thinking for each step of the solution process, both verbally and in writing. Teachers can provide a model as well as a list of guiding questions, such as "What do you notice about the structure of this problem?" and "How does that point you toward a particular strategy to solve it?"

Example prompts to encourage students to articulate their reasoning

- What did you notice first about the problem structure? How did that influence your solution strategy? What strategy is appropriate for solving this problem and why?
- What choices did you have to make in solving this problem?
- What goal were you trying to achieve?
- How did you get your answer? How do you know it is correct?
- Describe to another student how to solve this problem.
- What was most difficult about this problem? Did you run into any challenges? If so, what did you do to overcome them?

Note. Taken from Example 3.7 on page 33 in the practice guide.

3. Have students evaluate and compare different strategies for solving problems.

Instructional strategies from the examples

- Have students compare problem structures and solution strategies to discover relationships among similar and different problems, strategies, and/or solutions.
- Use solved problems showing two strategies side by side to enable students to see the number, type, and sequence of solution steps.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.1b, PS.1c, PS.1d, PS.3b, PS.3d, PS.7b, PS.7c
TEACHERS: INST.MS.2, INST.AM.4, INST.AM.5, INST.AM.7, INST.AM.9, INST.TH.2, INST.PS.1, PLAN.SW.1, PLAN.Desc.1

Once students have mastered a strategy, teachers should have them make comparisons across similar and different problem structures and strategies to identify relationships. Teachers should support students in considering how a solution strategy is similar to and different from others they have encountered. Teachers could encourage students to think about the accuracy, efficiency, and applicability of various problem-solving strategies. Guided discussion using worked problems can be helpful as students move from teacher-mediated to more individual work.

Example of small-group comparison and discussion activity

Objectives:

☑ Share and compare multiple solution strategies

☑ Use precise mathematical language to describe solution steps

☑ Explain reasoning and mathematical validity

Directions: Pair students off to work on algebra problems so that students with different strategies have the opportunity to talk with each other. For example, if two strategies are prevalent and approximately half of the students use each, students may be put into groups A and B based on like strategies and then each paired with a student from the other group. Partners can discuss the strategies they used to solve the first problem (e.g., What strategy did each person use? How did the strategies differ from one another? What was the partner’s rationale for using a different strategy? Did both strategies produce the same answer?). Challenge students to use their partner’s strategy when solving the next problem. Conclude the activity by asking students to reflect on what they discussed with their partners, explaining the most important ways in which the two strategies differ. Have students record the strategies discussed by the class.

Note. Taken from Example 3.9 on page 34 in the practice guide.

Potential roadblocks and how to address them

Roadblock	**Suggested Approach**
I'm worried about confusing my students by teaching them multiple strategies for solving a problem. They have a hard enough time learning one strategy! Isn't it easier for them to master one strategy for solving algebra problems?	Students are not expected to become experts in all strategies but to clarify their thinking when choosing the most appropriate one for a given problem. Teachers can focus on teaching one strategy at a time and then ask students to compare a new strategy with an established one. Different students may be more comfortable with certain strategies, so allowing them to explore multiple strategies will be helpful.
My special education students need a very structured process for solving algebra problems. Introducing multiple strategies and asking students to choose among strategies might be hard on them.	Teachers can provide explicit instruction to students with disabilities while still teaching them alternative strategies. Instruction should include both the steps and a clear rationale for application. Asking students to simply memorize a single strategy without building their understanding of how and why it is appropriate for a given problem type will lead to challenges for students in special education.
I can't seem to teach my students multiple strategies without them becoming confused. I presented and compared five algebra strategies for solving quadratic equations during one class period, and my students didn't understand. What should I do?	Students may not need diagrams to solve problems, but they will need them to notice and understand structural components. Make this explicit to students and continue to model the use of diagrams.

Roadblock	Suggested Approach
Teaching students to use and compare multiple strategies requires knowledge of many strategies and our textbook presents only one strategy.	The full practice guide provides lists of strategies for quadratic and linear system equations. Teachers can share strategies with one another in professional learning communities and create class posters or handouts for their students to access.
How can I stay on schedule teaching everything required to meet state standards and still have time to teach students to use multiple strategies?	Teachers can incorporate alternative and multiple strategies into existing lessons to develop students' critical thinking and algebraic reasoning. The focus should be on helping students reason algebraically and recognize when an alternative strategy might provide a solution that is more effective or efficient.

Reference: Star, J. R., Foegen, A., Larson, M. R., McCallum, W. G., Porath, J., & Zbiek, R. M. (2019). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2015-4010). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/20