This document provides a summary of Recommendation 1 from the WWC practice guide *Improving Mathematical Problem Solving in Grades 4 Through 8*. Full reference at the bottom of last page.

CONTENT: **Mathematics**
GRADE LEVEL(S): **4–8**
LEVEL OF EVIDENCE: **Minimal**

RECOMMENDATION

Prepare problems and use them in whole-class instruction.

Teachers should set aside time for problem-solving activities with the entire class instead of limiting problem-solving to individual homework assignments and include a variety of problems in these activities. Additionally, teachers should ensure that students understand the language, context, and math concepts of the problems included in lessons and homework.

HOW TO CARRY OUT THE RECOMMENDATION

1. Include both routine and nonroutine problems in problem-solving activities.

Instructional strategies from the examples

- Align use of routine or nonroutine problems with students' previous experience with problem-solving.
- Routine problems can be solved using familiar methods, replicating previously learned methods in a step-by-step fashion.

South Carolina standards alignment

MATHEMATICS: PS.1c
TEACHERS: INST.MS.1, INST.AM.4, INST.TCK.2, PLAN.SW.1

Nonroutine problems involve approaches that are not as predictable or well rehearsed; solution pathways aren't explicitly suggested by the task, task instructions, or in a worked-out example. Routine problems can be solved using approaches that students have already learned. Nonroutine problems, on the other hand, require using approaches that students are less familiar with or that are less

obvious from the problem. When the goal of a lesson is to help students understand the meaning of an operation or mathematical idea, teachers should select routine problems. These do not necessarily have to be simple—they can be complex, multistep problems that involve problem-solving approaches students are already familiar with. When the goal of a lesson is to develop students' ability to think strategically, teachers should select nonroutine problems.

Examples of routine problems

Likely routine for a student who has studied and practiced multiplication with mixed numbers:

Carlos is following a cookie recipe that calls for $1\,{}^{2}/_{3}$ cups of flour.
He needs to make 4 batches of cookies. How much flour does he need?
Likely routine for a student who has studied and practiced solving linear equations with one variable:

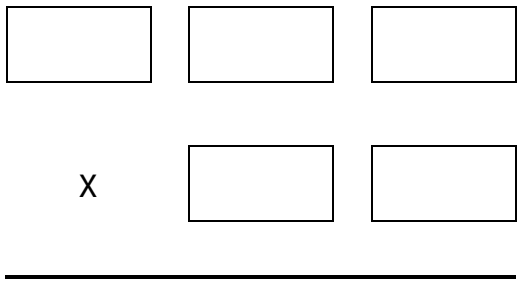
Solve for x: $20 + 8x = 60$

Note. Adapted from Example 1 on page 12 of the practice guide.

Examples of nonroutine problems

Likely nonroutine for students who are solidifying their understanding of multiplication:

The digits 1, 2, 3, 4, and 5—using each of these digits only once—are arranged in the blank boxes in the template below. Of all the possible arrangements of the digits 1 through 5, which one will produce the largest possible product?
Why does this arrangement work?



Likely nonroutine for students in beginning algebra:

There are 20 people in a room. Everybody shakes the hand of everyone else.
How many handshakes occurred?

Note. Adapted from Example 2 on page 12 of the practice guide.

2. Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.

Instructional strategies from the examples

- Explain contexts or vocabulary that may be unfamiliar to ensure students understand the language and context of problems—not to make problems less challenging, but to allow students to focus on the mathematics in the problem rather than on the need to learn new background knowledge or language.

South Carolina standards alignment

MATHEMATICS: PS.2d
TEACHERS: INST.PIC.2, INST.TCK.2, PLAN.Desc.1

The problems a teacher selects for a lesson may include unfamiliar vocabulary or contexts, making it challenging for students to focus on the math content. This is a particularly critical issue for English learners and students with disabilities. To ensure students' understanding without lessening the mathematical challenge, teachers can:

- Choose problems with language or contexts that are appropriate for the students' background.
- Clarify unfamiliar language or contexts in existing problems.
- Reword problems that contain unfamiliar words or phrases for students.

Examples of clarifying vocabulary and context

Example Problem	Vocabulary	Context
In a factory, 54,650 parts were made. When they were tested, 4% were found to be defective. How many parts were working?	Students need to understand the term defective as being the opposite of working and the symbol % as percent to correctly solve the problem.	What is a factory? What does parts mean in this context?

At a used-car dealership, a car was priced at $7,000. The salesperson then offered a discount of $350. What percent discount, applied to the original price, gives the offered price?	Students need to know what offered and original price mean to understand the goal of the problem, and they need to know what discount and percent discount mean to understand what mathematical operators to use.	What is a used-car dealership?

Note. Taken from Example 3 on page 14 of the practice guide.

3. Consider students' knowledge of math content when planning lessons.

Instructional strategies from the examples

- Review relevant skills and knowledge needed to understand and solve a problem, especially if the mathematical content has not been discussed recently or if a nonroutine, challenging problem is presented

South Carolina standards alignment

MATHEMATICS: PS.1c
TEACHERS: INST.MS.1, INST.AM.4, INST.TCK.2, PLAN.SW.1

Teachers should consider the concepts, skills, and vocabulary their students will need to solve problems included in lessons. For example, when finding the area of a circle, students may need to review the definitions of radius and pi as well as the concepts of perimeter and area. A brief review of the skills and vocabulary needed to understand and solve a problem may not only benefit struggling students but also help all students see how the knowledge they already have applies to more challenging problems.

Example of reviewing mathematical language

Problem	
Two vertices of a triangle are located at (0, 4) and (0, 10). The area of the triangle is 12 square units. What are all possible positions for the third vertex?	
Mathematical Language to Review	
• Vertices • Area square units	• Triangle • Vertex

Note. Taken from Example 5 on page 15 of the practice guide.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
Teachers are having trouble finding problems for the problem-solving activities.	Teachers can reference supplementary materials (for example, books on problem-solving), ask colleagues for additional problem-solving activities, or search the internet for examples. Useful resources on the internet include "Problems of the Week" from the Math Forum (https://www.nctm.org/pows/), "Illuminations" from the National Council of Teachers of Mathematics (https://illuminations.nctm.org/), and practice problems from standardized tests such as the PISA (http://www.oecd.org/pisa/; PISA 2012 Mathematics Items), SAT(https://collegereadiness.collegeboard.org/sat/practice), or TIMSS (TIMSS—Released Assessment Questions).
Teachers have no time to add problem-solving activities to their math instruction.	To make time during lessons, teachers can replace some of the problems students are required to solve during seatwork with fully solved problems that students can review and use as problem-solving models.
Teachers are not sure which words to teach when teaching problem-solving.	Math coaches and specialists can provide lists of words and phrases essential for teaching a given unit. Teachers can also work with colleagues to identify words students need to understand and solve problems. They can also look for important terms in class textbooks or state math standards.

Reference: Woodward, J., Beckman, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2018). *Improving mathematical problem solving in grades 4 through 8* (NCEE 2012-4055). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/16