This document provides a summary of Recommendation 2 from the WWC practice guide *Improving Mathematical Problem Solving in Grades 4 Through 8*. Full reference at the bottom of last page.

CONTENT: **Mathematics**
GRADE LEVEL(S): **4–8**
LEVEL OF EVIDENCE: **Strong**

RECOMMENDATION

Assist students in monitoring and reflecting on the problem-solving process.

Considering what students are doing and why they are doing it during the problem-solving process helps them learn math more effectively. It helps students assess the path they take while solving a problem and helps them connect knowledge they already have to newer concepts. Teachers can help students in this process through asking guiding questions, modeling self-monitoring and reflection, and building on students' own reflections to help them improve their problem-solving.

HOW TO CARRY OUT THE RECOMMENDATION

1. Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.

Instructional strategies from the examples

- Provide questions that students should ask and answer as they solve problems.
- Provide task lists that help students complete steps in the problem-solving process.
- Encourage students to explain and justify their response to each prompt, either orally or in writing.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c
TEACHERS: INST.AM.5, INST.TCK.2, INST.PS.1, PLAN.SW.1

Assist students in monitoring and reflecting on the problem-solving process.

Teachers can provide students with two types of prompts: (1) questions students should ask themselves, and (2) task lists that students can follow each time they work through solving a problem. When first introducing prompts, teachers may need to help students understand how to use them. Teachers can help students by talking through the questions or task lists in whole-class or small-group discussions. If students obtain an incorrect solution, teachers can provide the correct solution and ask students to explain why that answer is correct (and, conversely, why their original solution is wrong). The more competent students become at problem-solving, the less support teachers will need to give. Teachers should be sure not to overburden students with long lists of prompts. Doing so may lead students to solve problems more slowly or abandon the prompts altogether.

Example Questions	Example Task List
• What is this problem asking? • What do I know about the problem so far? What information is given to me? How can this help me? • Which information in this prompt is relevant to solving the problem? • What are some different ways I could approach solving this problem? • Why did this approach work? Why didn't it work?	1. Figure out what the question is asking for and what information is given. 2. Identify the type of problem. 3. Recall solutions to previous problems that may be useful in the current problem. 4. Make a visual to help represent or solve the problem. 5. Solve the problem. 6. Check your solution.

Note. Adapted from Examples 6 and 7 on page 19 of the practice guide.

2. Model how to monitor and reflect on the problem-solving process.

Instructional strategies from the examples

- As you work through a problem, say not only the response to each prompt but also the reason(s) why you took each.
- Alternatively, as you work a step in solving the problem, ask students to explain why this works.

South Carolina standards alignment

MATHEMATICS: PS.1c, PS.3d
TEACHERS: INST.PIC.3, INST.AM.5, INST.PS.1, PLAN.SW.1

Using prompts, teachers can show how to monitor and reflect during the problem-solving process. Teachers can give students an appropriate response to each prompt and either explain the reasoning behind that response or ask the students to explain why that response makes sense. Teachers should ensure that each step of the problem-solving process is represented by a prompt.

Example of modeling how to monitor and reflect

Problem
Last year was unusually dry in Colorado. Denver usually gets 60 inches of snow per year. Vail, which is up in the mountains, usually gets 350 inches of snow. Both places had 10 inches of snow less than the year before. Kara and Ramon live in Colorado and heard the weather report. Kara thinks the decline for Denver and Vail is the same. Ramon thinks that when you compare the two cities, the decline is different. Explain how both people are correct.

Solution

TEACHER: First, I ask myself, "What is this story about, and what do I need to find out?" I see that the problem has given me the usual amount of snowfall and the change in snowfall for each place, and that it talks about a decline in both cities. I know what decline means: "a change that makes something less." Now I wonder how the decline in snowfall for Denver and Vail can be the same for Kara and different for Ramon. I know that a decline of 10 inches in both cities is the same, so I guess that's what makes Kara correct. How is Ramon thinking about the problem?

I ask myself, "Have I ever seen a problem like this before?" As I think back to the assignments we had last week, I remember seeing a problem that asked us to calculate the discount on a $20 item that was on sale for $15. I remember we had to determine the percent change. This could be a similar kind of problem. This might be the way Ramon is thinking about the problem.

Before I go on, I ask myself, "What steps should I take to solve this problem?" It looks like I need to divide the change amount by the original amount to find the percent change in snowfall for both Denver and Vail.

Denver: $10 \div 60 = 0.166$ or 16.67% or 17% when we round it to the nearest whole number

Vail: $10 \div 350 = 0.029$ or 2.9% or 3% when we round it to the nearest whole number

So the percent decrease in snow for Denver was much greater (17%) than for Vail (3%). Now I see what Ramon is saying! It's different because the percent decrease for Vail is much smaller than it is for Denver.

Finally, I ask myself, "Does this answer make sense when I reread the problem?" Kara's answer makes sense because both cities did have a decline of 10 inches of snow. Ramon is also right because the percent decrease for Vail is much smaller than it is for Denver. Now, both of their answers make sense to me.

Note. Taken from Example 8 on page 20 of the practice guide.

3. Use student thinking about a problem to develop students' ability to monitor and reflect.

Instructional strategies from the examples

- Help students verbalize other ways to think about the problem.
- Include guided questioning to help students clarify and refine their thinking or establish a method for monitoring and reflecting that makes sense to them.

South Carolina standards alignment

MATHEMATICS: PS.1a, PS.1c, PS.3a
TEACHERS: INST.MS.1, INST.MS.2, INST.AM.5, INST.PS.1, PLAN.SW.1, PLAN.Desc.1

Teachers can help students establish methods for monitoring and reflecting that make sense to students. Teachers can establish a dialogue with students that includes guiding questions to clarify and refine their thinking. This activity is helpful for students who dislike, or have trouble understanding, teacher-provided prompts.

Example of using student ideas to clarify and refine the monitoring and reflecting process

Problem
Find a set of five different numbers whose average is 15.
Solution
TEACHER: Jennie, what did you try? **STUDENT:** I'm guessing and checking. I tried 6, 12, 16, 20, 25, and they didn't work. The average is like 17.8 or something decimal like that. **TEACHER:** That's pretty close to 15, though. Why'd you try those numbers? **STUDENT:** What do you mean? **TEACHER:** I mean, where was the target, 15, in your planning? It seems like it was in your thinking somewhere. If I were choosing five numbers, I might go with 16, 17, 20, 25, 28. **STUDENT:** But they wouldn't work—you can tell right away.

TEACHER: How?

STUDENT: Because they are all bigger than 15.

TEACHER: So?

STUDENT: Well, then the average is going to be bigger than 15.

TEACHER: Okay. That's what I meant when I asked, "Where was 15 in your planning?" You knew they couldn't all be bigger than 15. Or they couldn't all be smaller either?

STUDENT: Right.

TEACHER: Okay, so keep the target, 15, in your planning. How do you think five numbers whose average is 15 relate to the number 15?

STUDENT: Well, some have to be bigger and some smaller. I guess that's why I tried the five numbers I did.

TEACHER: That's what I guess, too. So the next step is to think about how much bigger some have to be and how much smaller the others have to be. Okay?

STUDENT: Yeah.

TEACHER: So use that thinking to come up with five numbers that work.

Note. Taken from Example 9 on page 21 of the practice guide.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
Students don't want to monitor and reflect; they just want to solve the problem.	Explain to students that getting into the habit of monitoring and reflecting every time they solve a problem will improve their problem-solving abilities, and that monitoring and reflecting are still an integral part of the process for experienced problem-solvers.
Teachers are unclear on how to think aloud while solving a nonroutine problem.	Teachers can prepare ahead of time by creating outlines of responses to prompts and by anticipating how students may think about prompts. They can seek help from colleagues or math coaches if they get stuck.
Students take too much time to monitor and reflect on the problem-solving process.	While students may solve problems slowly when they begin learning how to monitor and reflect, they will become more efficient with practice.
When students reflect on the problems they have already solved, they resort to using methods from problems rather than adapting their efforts to the new problem before them.	Ask students to explain why their solution worked in the previous problem, and why it may or may not work for the current problem.

Reference: Woodward, J., Beckman, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2018). *Improving mathematical problem solving in grades 4 through 8* (NCEE 2012-4055). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/16