This document provides a summary of Recommendation 3 from the WWC practice guide *Improving Mathematical Problem Solving in Grades 4 Through 8*. Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **4–8**

LEVEL OF EVIDENCE: **Strong**

Recommendation

Teach students how to use visual representations.

Translating quantitative information into an algebraic or arithmetic form is a critical component of the problem-solving process. Students who learn how to represent this information visually before translating it into an algebraic or arithmetic form tend to be more effective at problem-solving. When teaching students to use visual representations (e.g., graphs, diagrams, number lines, tables), teachers should choose visuals that are appropriate for the problem at hand and their students, then use them consistently for similar problems so as not to overwhelm students with too many differing examples.

Sample visual representations

Strip diagrams use rectangles to represent quantities presented in the problem.

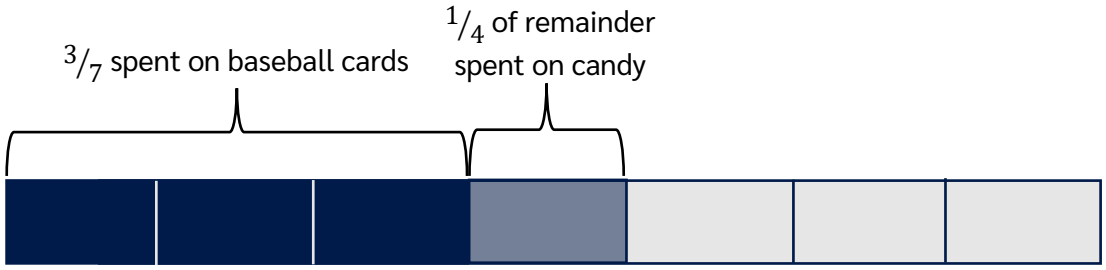
Percent bars are strip diagrams in which each rectangle represents a part of 100 in the problem.

Schematic diagrams demonstrate the relative sizes and relationships between quantities in the problem.

Below are two examples of how visual representations might be used to solve problems. For additional examples, see page 24 of the practice guide.

Problem
Ben spent $^3/_7$ of his allowance on baseball cards and then $^1/_4$ of what remained on candy. After this, he had $50 left. How much did he start with?

Sample Strip Diagram



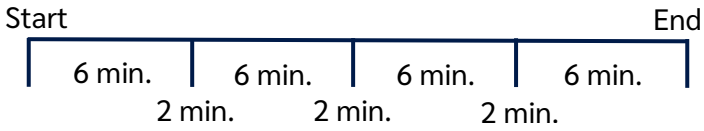
This diagram depicts the original amount of money that Ben had, divided into 7 equal parts. It can be seen that 3 of the 7 parts have not been spent. From this diagram, there are multiple approaches to creating an equation.

Note. Adapted from Example 10 on page 24 of the practice guide.

Problem

Jackie usually jogs 4 laps around a track, and each lap takes 6 minutes.
Because of injury, she needs to rest for 2 minutes between each lap.
How long does it take her to complete the 4 laps?

Sample Schematic Diagram



This diagram illustrates Jackie running 4 laps, with each lap taking 6 minutes. As she runs, Jackie needs to take a 2-minute break between each lap. From this diagram, an equation, such as $(6 \times 4) + (2 \times 3) = x$, can be created to find the total number of minutes Jackie takes to run 4 laps.

Note. Adapted from Example 10 on page 25 of the practice guide.

How to carry out the recommendation

1. Select visual representations that are appropriate for students and the problems they are solving.

Instructional strategies from the examples

- Use schematic diagrams with ratio and proportion problems.
- Use percent bars for percent problems
- Use strip diagrams for comparison and fraction problems.

South Carolina standards alignment

MATHEMATICS: PS.2b
TEACHERS: INST.PIC.2, INST.AM.6, INST.TCK.2

Rather than using all visual representations recommended for a particular type of problem, teachers should select the visual representations they think will work best for their students. Teachers should use selected representations consistently for similar problems so as not to overwhelm students with too many examples and give students time to learn how to successfully use the selected representations. If students still struggle with a representation after a reasonable amount of time, teachers should consider using a different type of representation.

2. Use think-alouds and discussions to teach students how to represent problems visually.

Instructional strategies from the examples

- Demonstrate how to represent the problem using the representation, using think-alouds to describe the decisions you make to connect the problem to the representation.
- Explain how to identify the problem type based on mathematical ideas in the problem and why a certain representation is most appropriate.
- Teach students to identify what information is relevant or critical to solving the problem.
- Encourage students to discuss similarities or differences among visuals they have used.

South Carolina standards alignment

MATHEMATICS: PS.1c, PS.2b, PS.4a, PS.7c
TEACHERS: INST.MS.2, INST.PIC.2, INST.PIC.3, INST.TCK.2, INST.PS.

Teachers should demonstrate the thought process behind connecting a problem to a visual representation by thinking aloud when explaining a new representation. Thinking aloud goes beyond teachers telling students what they are doing; it involves teachers explaining why they are taking the particular steps. Teachers should explain how they identified the type of math problem and why they think the selected representation is appropriate for that problem. They should demonstrate how to identify the information in a problem that is relevant to solving it.

Example of using a think-aloud

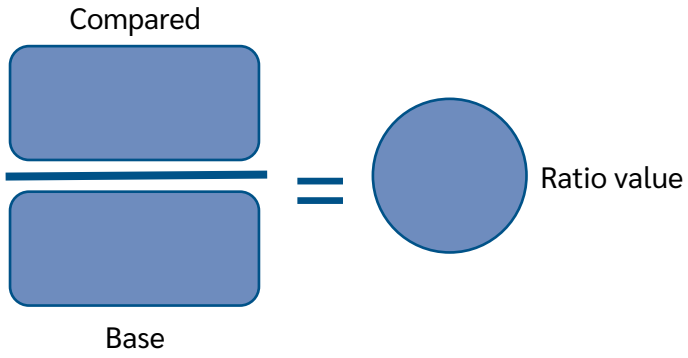
Problem
Monica and Bianca went to a flower shop to buy some roses. Bianca bought a bouquet with 5 pink roses. Monica bought a bouquet with two dozen roses, some red and some yellow. She has 3 red roses in her bouquet for every 5 yellow roses. How many red roses are in Monica's bouquet?

Solution

TEACHER: I know this is a ratio problem because two quantities are being compared: the number of red roses and the number of yellow roses. I also know the ratio of the two quantities. There are 3 red roses for every 5 yellow roses. This tells me I can find more of each kind of rose by multiplying.

I reread the problem and determine that I need to solve the question posed in the last sentence: "How many red roses are in Monica's bouquet?" Because the question is about Monica, perhaps I don't need the information about Bianca. The third sentence says there are two dozen red and yellow roses. I know that makes 24 red and yellow roses, but I still don't know how many red roses there are. I know there are 3 red roses for every 5 yellow roses. I think I need to figure out how many red roses there are in the 24 red and yellow roses.

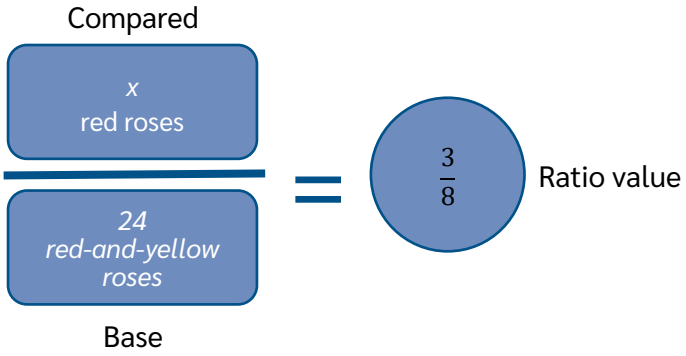
Let me reread the problem . . . That's correct. I need to find out how many red roses are in the bouquet of 24 red and yellow roses. The next part of the problem talks about the ratio of red roses to red and yellow roses. I can draw a diagram that helps me understand the problem. I've done this before with ratio problems. These kinds of diagrams show the relationship between the two quantities in the ratio.



TEACHER: I write the quantities and units from the problem and an x for what must be solved in the diagram. First, I am going to write the ratio of red roses to yellow roses here in the circle. This is a part-to-whole comparison—but how can I find the whole in the part-to-whole ratio when we only know the part-to-part ratio (the number of red roses to the number of yellow roses)?

I have to figure out what the ratio is of red roses to red and yellow roses when the problem only tells me about the ratio of red roses to yellow roses, which is 3:5. So if there are 3 red roses for every 5 yellow roses, then the total number of units for red and yellow roses is 8. For every 3 units of red roses, there are 8 units of red

and yellow roses, which gives me the ratio 3:8. I will write that in the diagram as the ratio value of red roses to red and yellow roses. There are two dozen red and yellow roses, and that equals 24 red and yellow roses, which is the base quantity. I need to find out how many red roses (x) there are in 24 red and yellow roses.



I can now translate the information in this diagram to an equation like this:

$$\frac{x\ red\ roses}{24\ red\text{–}and\text{–}yellow\ roses} = \frac{3}{8}$$

Then, I need to solve for x.

$$\frac{x}{24} = \frac{3}{8}$$

$$24\left(\frac{x}{24}\right) = 24\left(\frac{3}{8}\right)$$

$$x = \frac{72}{8}$$

$$x = 9$$

Note. Taken from Example 11 on pages 27–28 of the practice guide.

3. Show students how to convert the visually represented information into mathematical notation.

Instructional strategies from the examples

- Show students how each quantity and relationship in the visual representation corresponds to those in the equation.

South Carolina standards alignment

MATHEMATICS: PS.1c, PS.2a, PS.2b, PS.4a, PS.4b
TEACHERS: INST.PIC.2, INST.TCK.2

Teachers should show students how to translate quantities and relationships in visual representations into math equations. Sometimes, all this translation requires is removing boxes, arrows, and other visual elements from the representation. With more complicated examples, teachers may need to provide more explicit illustrations of the connection between the representations and mathematical notation.

Potential roadblocks and how to address them

Roadblock	Suggested **Approach**
Students do not capture the relevant details in the problem or include unnecessary details when representing a problem visually.	If students are missing relevant details in their visual representations, teachers can ask guiding questions to build on students' thinking and refine their representations. Once the representations are refined, teachers can ask students why their initial representations did not work. If guiding questions do not work, teachers can demonstrate how to alter students' representations to represent the problems appropriately and eliminate unnecessary detail. Teachers should point out elements of the representations that were done correctly so students are encouraged to continue trying.
The class text does not use visual representations.	Teachers can incorporate visual representations into lessons using media such as whiteboards, overhead projectors, or interactive whiteboards. Teachers can tap colleagues or math coaches for useful visual representations or develop their own. The internet and professional development materials may have useful examples.

Reference: Woodward, J., Beckman, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2018). *Improving mathematical problem solving in grades 4 through 8* (NCEE 2012-4055). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/16