This document provides a summary of Recommendation 4 from the WWC practice guide *Improving Mathematical Problem Solving in Grades 4 Through 8*. Full reference at the bottom of last page.

CONTENT: **Mathematics**

GRADE LEVEL(S): **4–8**

LEVEL OF EVIDENCE: **Moderate**

Recommendation

Expose students to multiple problem-solving strategies.

Students who learn how to use multiple problem-solving strategies can approach problems with greater ease and flexibility in finding solutions. Teachers should demonstrate that problems can be solved in multiple ways and that approaches to problem-solving should be selected for their ease and efficiency. Teachers can present students with a number of problem-solving strategies and then give them the opportunity to compare, contrast, and carry out the strategies.

How to carry out the recommendation

1. Provide instruction in multiple strategies.

Instructional strategies from the examples

- Demonstrate multiple ways to solve the same problem.
- At times, use an unsuccessful strategy and demonstrate changing to an alternate strategy.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c
TEACHERS: INST.PIC.2, INST.AM.9, INST.TCK.2, INST.PS.1

Teachers can present strategies for general use as well as those for specific problems. When demonstrating different strategies, teachers should occasionally attempt unsuccessful strategies and then change to successful ones. Doing so will show students that some problems may not be easy to solve the first time and they may need to try more than one strategy before successfully solving a problem. Teachers should also demonstrate more than one successful approach to the same problem.

Example of two ways to solve the same problem

Problem

Ramona's furniture store has a choice of 3-legged stools and 4-legged stools. There are five more 3-legged stools than 4-legged stools. When you count the legs of the stools, there are exactly 29 legs. How many 3-legged and 4-legged stools are there in the store?

Solution 1: Guess and Check

4 x 4 legs = 16 legs	9 x 3 legs = 27 legs	Total = 43 legs
3 x 4 legs = 12 legs	8 x 3 legs = 24 legs	Total = 36 legs
2 x 4 legs = 8 legs	7 x 3 legs = 21 legs	Total = 29 legs

TEACHER: This works; the total equals 29, and with two 4-legged stools and seven 3-legged stools, there are five more 3-legged stools than 4-legged stools.

Solution 2

TEACHER: Let's see if we can solve this problem logically. The problem says that there are five more 3-legged stools than 4-legged stools. It also says that there are 29 legs altogether. If there are five more 3-legged stools, there has to be at least one 4-legged stool in the first place. Let's see what that looks like.

Stools	(1 four-legged stool)		(6 three-legged stools)
Total legs	$4 \times 1 = 4$	+	$3 \times 6 = 18$
		$4 + 18 = 22$	

TEACHER: We can add a stool to each group, and there will still be a difference of five stools.

Stools	(2 four-legged stools)		(7 three-legged stools)
Total legs	$4 \times 2 = 8$	+	$3 \times 7 = 21$
		$8 + 21 = 29$	

TEACHER: I think this works. We have a total of 29 legs, and there are still five more 3-legged stools than 4-legged stools. We solved this by thinking about it logically. We knew there was at least one 4-legged stool and there were six 3-legged stools. Then we added to both sides so we always had a difference of five stools.

Note. Taken from Example 14 on page 34 of the practice guide.

2. Provide opportunities for students to compare multiple strategies in worked examples.

Instructional strategies from the examples

- Ask students to compare the similarities and differences among multiple strategies.
- Provide opportunities for students to work with a partner to discuss strategies in worked examples.
- Use worked examples alongside opportunities for students to solve problems on their own.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c, PS.3b, PS.3d, PS.7b, PS.7c
TEACHERS: INST.MS.2, INST.AM.4, INST.AM.7, INST.AM.9, INST.TCK.2, INST.TH.2, INST.PS.1, PLAN.SW.1

Teachers should provide side-by-side examples of different problem-solving strategies and give students the opportunity to work together to compare the strategies. Teachers can prompt students to compare and contrast the strategies, to justify the approach they would choose, and to consider why two different approaches can lead to the same answer. Teachers can provide worked examples alongside problems that students are required to solve. Students can practice describing their solution paths both verbally and in writing.

Example of comparing strategies

Sanjin's Solution		Emily's Solution	
$7(x-3)=4(x-3)-3$		$7(x-3)=4(x-3)-3$	
$7x-21=4x-12-3$	Distribute	$3(x-3)=-3$	Subtract on both
$7x-21=4x-15$	Combine	$x-3=-1$	Divide on both
$3x-21=-15$	Subtract on both	$x=2$	Add on both
$3x=6$	Add on both		
$x=2$	Divide on both		
TEACHER: Sanjin and Emily used different approaches but got the same answer. Why is this? Which of their approaches would you choose? Why?			

Note. Adapted from Example 15 on page 35 of the practice guide.

3. Ask students to generate and share multiple strategies for solving a problem.

Instructional strategies from the examples

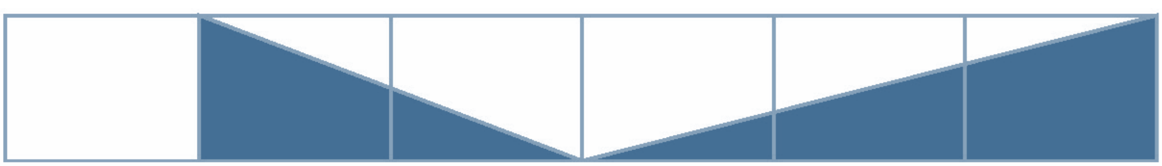
- Rather than randomly calling on students to share their strategies, select students purposefully based on the strategies they have used to solve the problem.

South Carolina standards alignment

MATHEMATICS: PS.1b, PS.1c, PS.3a, PS.5a, PS.7b, PS.7c
TEACHERS: INST.MS.1, INST.MS.2, INST.AM.4, INST.AM.7, INST.AM.9, INST.TH.2, INST.PS.1, PLAN.SW.1, PLAN.Desc.1

Teachers should encourage students to generate multiple problem-solving strategies and give them opportunities to share their strategies with the class. Rather than calling on students randomly, teachers should call on students who generated strategies different from the one presented. Teachers should encourage students to not only present their approach but also to explain why they chose that approach. Examples 16 and 17 on pages 36–37 of the practice guide provide excellent models for teachers.

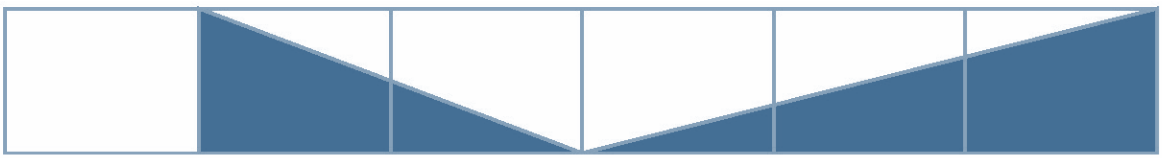
Example of two students sharing their strategies for solving a fractions problem

Problem
What fraction of the whole rectangle is blue? 

Solution

STUDENT 1: If I think of it as what's to the left of the middle plus what's to the right of the middle, then I see that on the left, the blue part is $^1/_3$ of the area, so that is $^1/_3$ of $^1/_2$ of the entire rectangle. On the right, the blue part is $^1/_2$ of the area, so it is $^1/_2$ of $^1/_2$ of the entire rectangle. This information tells me that the blue part is

$(^1/_3 \times {^1/_2}) + (^1/_2 \times {^1/_2}) = {^1/_6} + {^1/_4} = {^2/_{12}} + {^3/_{12}} = {^5/_{12}}$ of the entire rectangle.



STUDENT 2: I see that the original blue part and the part I've colored dark blue have the same area. So the original blue part is $^1/_2$ of the dark-blue-and-blue part, or $^1/_2$ of $^5/_6$ of the entire rectangle.

This tells me that the original blue part is $^1/_2 \times {^5/_6} = {^5/_{12}}$ of the entire rectangle.

Note. Taken from example 17 on page 37 of the practice guide.

Potential roadblocks and how to address them

Roadblock	***Suggested Approach***
Teachers don't have enough time in their math class for students to present and discuss multiple strategies.	Teachers can ask students to write out their strategies on personal whiteboards or chart paper so they do not have to spend time rewriting them on the class whiteboard. Another approach is for teachers to document the strategies students come up with during independent or small-group work and summarize them for the class.
Not all students are willing to share their strategies.	Teachers should encourage students to share their strategies even if they are incorrect. They should explain to students that there may be a variety of approaches to solving most problems and that the solutions they share may not have been considered by the other students. Teachers can point out that sharing will help students learn effective problem-solving strategies from one another.
Some students struggle to learn multiple strategies.	If students lack or cannot retrieve necessary knowledge, they may struggle with using multiple strategies. Teachers may need to ask students to write down the facts of a problem before trying to solve it. They may need to modify a problem to make it easier to focus on problem-solving (rather than the arithmetic, for example). Teachers can also write down problem solutions side by side so that students can more easily compare the solutions.
Some of the strategies students share are not clear or do not make sense to the class.	Teachers can walk around the classroom and ask students to explain their approaches individually. Doing so will better prepare teachers to clarify students' thinking when they share their approaches with the class, either by asking guiding questions or by rewording the students' approaches. Teachers can also ask other students to restate what a student said.

Reference: Woodward, J., Beckman, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2018). *Improving mathematical problem solving in grades 4 through 8* (NCEE 2012-4055). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://ies.ed.gov/ncee/wwc/PracticeGuide/16