

This document provides a summary of Recommendation 1 from the WWC practice guide *Teaching Math to Young Children*. Full reference at the bottom of last page.

CONTENT: Mathematics

GRADE LEVEL(S): preK–K

LEVEL OF EVIDENCE: Moderate

Recommendation

Teach numbers and operations using a developmental progression.

Understanding what skills and knowledge children already possess is the starting point for instruction. A developmental progression can provide a road map of the next steps. For example, when teaching numbers and operations, teachers should ensure that children proceed through each level of the developmental progression. At each level, children should begin by practicing with small sets of objects and progress to larger sets until they master the skills and knowledge at that level.

Example of a piece of developmental progression for number knowledge

Subitizing (small-number recognition)	Subitizing refers to a child’s ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as non-examples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three.
	A child is ready for the next step when , for example, they can see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful object counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle).
	A child is ready for the next step when , for example, if given five blocks and asked, “How many?” they count by pointing and assigning one number to each block: “One, two, three, four, five,” and recognizes that the total is “five.”

Counting-based comparisons of collections larger than three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., “seven” items is more than “six” items because you have to count further).
	A child is ready for the next step when they are shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., “nine” bears is more).
Number-after knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one.
	A child is ready for the next step when they can answer questions such as, “What comes after five?” by stating “five, six” or simply “six” instead of, say, counting “one, two, . . . six.”
Mental comparisons of close or neighboring numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that “nine” is larger than “eight”).
	A child has this knowledge when they can answer questions such as, “Which is more, seven or eight?” and can make comparisons of other close numbers.
Number-after equals one more	Once children can mentally compare numbers and see that “two” is one more than “one” and that “three” is one more than “two,” they can conclude that any number in the counting sequence is exactly one more than the previous numbers.
	A child is ready for the next step when they recognize, for example, that “eight” is one more than “seven.”

Note. Taken from Table 3 on page 13 of the practice guide.

How to carry out the recommendation

1. First, provide opportunities for children to practice recognizing the total number of objects in small collections (one to three items) and labeling them with a number word without needing to count them.

Instructional strategies from the examples

- Build children’s understanding of quantity through subitizing activities.
- Have children identify sets that contain an equal number of objects without counting.
- Build children’s ability to compare sets by having them find sets of objects that contain the same number of objects and those that do not (e.g., “three” and “not three”).

South Carolina standards alignment

MATHEMATICS: K.NS.6

TEACHERS: No direct alignment

Children should be able to determine the number of objects in a small set without counting. This is known as *subitizing*. During classroom transitions, teachers may find small sets of objects (five or less) in the classroom and ask, “How many (object name) do you see?” After children can successfully identify three objects, they should be able to understand related sets that have the same number of objects. For example, three pencils and three erasers are two sets with the same number of objects. Once children have experience in recognizing sets of objects containing similar items, teachers can progress to sets with dissimilar items (for example, a set of three containing a pencil, a crayon, and an eraser). When developing subitizing, children may overgeneralize the term “three” or “four” to mean “many,” so teachers should identify a set of three objects as “three” and a set of four objects as “not three” to help children recognize the difference. Teachers can then challenge children to find sets of objects around the classroom that are “three” or “not three.” See Example 1 on page 16 of the practice guide referenced on the last page of this document.

2. Next, promote accurate one-to-one counting as a means of identifying the total number of items in a collection.

Instructional strategies from the examples

- Build on children’s ability to subitize to help them develop one-to-one object counting, beginning with organized sets, then moving to unorganized sets.
- After successfully counting a set of objects, help children develop cardinality—recognizing that the last counting word represents the number of objects in the set.
- Capitalize on children’s counting errors to strengthen their counting ability.

South Carolina standards alignment

MATHEMATICS: K.NS.5

TEACHERS: PLAN.SW.3

One-to-one counting occurs when children count with number words in a consecutive order to determine the number of objects in a set, using only one number name for each unique item. For example, a child counting a set of pens points at a pen and says, “One,” then points at the next pen and says, “Two,” and finally points at the last pen and says, “Three.” Children should begin with small sets (one to four objects). Teachers should help them in realizing that the last number they count is the total number in a set. Then, children can progress to larger sets (four to 10 objects). Teachers should also demonstrate that order does not affect the result by using objects around the classroom and counting them in different ways. For example, when counting pens, no matter which pen children start with, they will reach the same result.

Example counting activity: The *Hidden Stars* game

Objective
Practice using one-to-one counting and the final number counted to identify “how many” objects.
Materials needed: <ul style="list-style-type: none">• Star stickers in varying quantities from one to 10, glued to 5-by-8-inch cards• Paper for covering cards
Directions: Teachers can tailor the <i>Hidden Stars</i> game for use with the whole class, a small group, or individual children. Show children a collection of stars on an index

card. Have one child count the stars. Once the child has counted the stars correctly, cover the stars and ask, “How many stars am I hiding?”

Early math content areas covered

- Counting
- Cardinality (using the last number counted to identify the total in the set)

Monitoring children’s progress and tailoring the activity appropriately

- Work with children in a small group, noting each child’s ability to count the stars with accuracy and say the amount using the cardinality principle (the last number counted represents the total).
- When children repeat the full count sequence, model the cardinality principle. For example, for four items, if a child repeats the count sequence, say, “One, two, three, four. So I need to remember four. There are four stars hiding.”
- Have a child hide the stars while telling him or her how many there are, emphasizing the last number as the significant number.

Using the activity to increase math talk in the classroom

- Ask, “How many?” (e.g., “How many blocks did you use to build your house? How many children completed the puzzle?”)

Note. Taken from Example 2 on page 18 of the practice guide.

Errors are to be expected when children learn how to count. Common counting errors include sequence errors, coordination errors, and keeping-track errors, as well as skimming and not recognizing cardinality. In sequence errors, children confuse the sequence of numbers. In this case, teachers might ask the children to sing the number sequence. If children are skipping specific numbers, teachers should focus on practicing that part of the sequence. Coordination errors involve children labeling an object with more than one word or pointing to an object without counting it. Teachers might correct these errors by encouraging children to slow down and count each object once. When children make keeping-track errors, they may count an object twice. Teachers can help children differentiate counted objects from uncounted objects by having them pick up the counted objects and place them to the side. For more information on common counting errors and recommendations on how to remedy them, see Table 4 on page 19 of the practice guide referenced on the last page of this document.

3. Once children can recognize or count collections, provide opportunities for children to use number words and counting to compare quantities.

Instructional strategies from the examples

- Initially present children with sets in which one is obviously larger, then move to sets that are more equal in size that require counting or subitizing.

South Carolina standards alignment

MATHEMATICS: K.NS.7

TEACHERS: PLAN.IP.3, PLAN.SW.3

Children can progress in making meaningful comparisons of sets by, for example, identifying “more” and “fewer.” First, teachers should present children with two sets of objects, one of which is obviously larger, and ask them which set has more or fewer objects. Next, teachers can demonstrate that the further in the counting sequence children count, the larger the numbers become. The sample chart below provides a visual of this increase in size or quantity and can be helpful. Real-world examples, such as counting to determine who has more points in a game, can also help children increase their ability to compare quantities. Also helpful is knowing what number comes next without counting (number-after relations). Teachers can reinforce this knowledge by asking children such questions as “This is the sixth pen, so the next pen will be how many?”

Sample cardinality chart

									●
								●	●
							●	●	●
						●	●	●	●
				●	●	●	●	●	●
			●	●	●	●	●	●	●
		●	●	●	●	●	●	●	●
	●	●	●	●	●	●	●	●	●
●	●	●	●	●	●	●	●	●	●
1	2	3	4	5	6	7	8	9	10

Note. Taken from page 20 of the practice guide.

4. Encourage children to label collections with number words and numerals.

Instructional strategies from the examples

- Label sets of objects with representations of quantity, numeral, and number words to help children connect the different representations.

South Carolina standards alignment

MATHEMATICS: K.NS.4a

TEACHERS: PLAN.SW.3

Numerals are a way to represent quantity. To help children understand this, teachers should pair number words with numerals. For example, “3,” “three,” and “three objects” have the same meaning. Teachers can label sets of objects that children see in the classroom. For example, a set of three pens can be labeled “3, •••, three.” The three dots provide scaffolding and represent the quantity for children who do not recognize numerals yet.

Example activity: The *Concentration: Numerals and Dots* game

Objective
<p>Match numerals with corresponding quantities.</p> <p>Materials needed:</p> <ul style="list-style-type: none">• One set of 20 cards: 10 cards with numerals from 1 to 10 along with the corresponding number of dots, and 10 cards with pictures of objects (the numbers of objects corresponding to a numeral from 1 through 10).• For even more advanced play, once children are proficient at numerals 1–10, teachers can create cards for numerals 11–20. <p>Directions: Half of the cards have a numeral and dots to represent the amount (e.g., the numeral 3 and three dots) on one side, and the other half have pictures of collections of objects on one side (e.g., three horses, four ducks). The other side of each card is blank. The cards are placed facedown, with the numeral cards in one area and the picture cards in another. A player chooses one numeral card and one picture card. If they match, then the player keeps those cards. Play continues until no further matching cards remain. The player with the most cards wins the game.</p>
Early math content areas covered
<ul style="list-style-type: none">• Numeral recognition.

- Corresponding quantity.
- If the objects in the pictures on the cards are in different orders, it can help reinforce the idea that appearance does not matter when it comes to numbers.

Monitoring children's progress and tailoring the activity appropriately

- Play the game with a small group of children, noting each child's progress in practicing and achieving the objectives.
- This game can be played with children who are not familiar with numeracy concepts.
- Use fewer cards, lower numbers, or cards with dots to scaffold. As children gain proficiency with the concepts, increase the number of cards and the size of the numbers.

Using the activity to increase math talk in the classroom

- Before asking, "How many?" ask, "How can we find out how many?"

Note. Taken from Example 3 on page 22 of the practice guide.

5. Once children develop these fundamental number skills, encourage them to solve basic problems.

Instructional strategies from the examples

- Provide opportunities for children to explore the effects of adding or removing one object from a set using counting strategies.
- Once children have developed some facility, present situations where the final result is hidden.

South Carolina standards alignment

MATHEMATICS: K.PS.1, K.PS.4, K.ATO

TEACHERS: PLAN.IP.3, PLAN.SW.3

Children should proceed to develop an understanding of the effects of altering the number of objects in a set as they prepare for math problem-solving. To start, children can remove or add one object, recount, and review how the number of objects has changed. They can also use counting strategies in problem-solving activities in the classroom, such as counting the number of groups to determine how many whiteboards to hand out.

After children have had opportunities to explore adding and subtracting objects from sets, teachers can move to situations where the final results are hidden from view. For example, teachers can show children a set of four pens, cover the pens with a cloth, take one pen from underneath the cloth, and then ask the children to determine how many pens are left under the cloth. Once the children decide how many objects are left, teachers can remove the cloth and have the children count to see if they solved the problem correctly. Snack time is also a great opportunity for children to apply counting skills. Teachers can ask, “How many will you have after you eat one snack item?” or, “How many will you have after your friend gives you one snack item?” to have children problem-solve without seeing the end set.

Potential roadblocks and how to address them

Roadblock	Suggested Approach
<i>I want to provide strong math foundations for my children, but I am not really comfortable with math myself.</i>	Teachers who feel less comfortable with math should base classroom projects on real-world examples. Setting up a toy store in the classroom provides a more comfortable setting in which to integrate lessons. Any activity that is of interest and involves counting presents an opportunity to build children's math skills.
<i>Each child in the class is at a different level in the developmental progression I am using to guide instruction.</i>	Teachers might split children into groups, using the developmental progression to create groups of children at a similar level. Dividing children into groups allows teachers to assign tasks based on each group's level of proficiency. Teachers can also create groups containing children at diverse levels of proficiency. This allows for children at a higher level to model a skill for others.
<i>A child is stuck at a particular point in the developmental progression.</i>	If children are stuck, they have likely not yet mastered a skill from an earlier level in the developmental progression. Teachers should use the developmental progression to help identify the unmastered skill and provide opportunities for children to practice it further before returning to the point at which they were stuck.

Reference: Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). *Teaching math to young children* (NCEE 2014-4005). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance.
<https://ies.ed.gov/ncee/wwc/PracticeGuide/18>